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On the superstability of a special derivation

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Abstract. The aim of this paper is to show that under some mild conditions a functional equation of multiplicative (α, β)-derivation is superstable on standard operator algebras. Furthermore, we prove that this generalized derivation can be a continuous and an inner (α, β)- derivation.

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1. Introduction

Questions concerning the stability of functional equations seems to be originated with S. M. Ulam [15]. In fact if X and Y are two Banach spaces and if $f : X \to Y$ is an approximately additive mapping, he wanted the functional equation for additive functions to be stable.

The case of approximately additive mapping between Banach spaces was solved by D. H. Hyers [9]. In 1968 S. M. Ulam proposed a more general problems: " When is it true that by changing the hypothesis of Hyers theorem a little one can still assert that the thesis of the theorem remains true of approximately true!"

Th. M. Rassias [12] proved a substantial generalization of the result of Hyers. Taking it into account, the additive functional equation is said to have the "Hyers- Ulam- Rassias" stability. And many authors answered the Ulam's equation for several cases. In [4] the

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author proved that every mapping f of a Banach algebra A onto a Banach algebra B which is approximately multiplicative is a ring homomorphism of A onto B.
J. A. Baker [2] showed that every approximately multiplicative unbounded complex-valued function defined on a semigroup S is actually a multiplicative function. We can find further references on problems concerning stability and superstability in survey papers .
In 1994 Peter Semrl [14] proved that the question of multiplicative derivation is superstable on standard operator algebras. In this paper at first we prove that the functional equation of linear (α, β)-derivation is superstable on whole of $\mathcal{B}(X)$ and furthermore we prove that this (α, β)-derivation can be a continuous and inner (α, β)-derivation and then we extend superstability to functional equation of multiplicative (α, β)-derivation on standard operator algebras. (for further results see [3],[9],[16]).

2. Preliminaries

Let R and S be two arbitrary associative rings (not necessarily with identity element). A mapping $\sigma : R \longrightarrow S$ such that $\sigma(x+y) = \sigma(x) + \sigma(y) \quad (x, y \in R)$ is called an additive mapping of R into S and is called a multiplicative mapping of R into S if $\sigma(xy) = \sigma(x)\sigma(y) \quad (x, y \in R)$ and a ring homomorphism from R into S is a mapping that is additive and also multiplicative. Furthermore a one to one and onto ring homomorphism is called a ring isomorphism and a ring isomorphism from R into R is called a ring automorphism of R.
If α and β are mappings on R, by multiplicative derivation from R into itself we call a mapping $D : R \longrightarrow R$ such that

$$D(xy) = D(x)y + xD(y) \quad (x, y \in R).$$

And by a multiplicative $(\alpha, \beta) - \textit{derivation}$ from R into itself we call a mapping $D : R \longrightarrow R$ such that

$$D(xy) = D(x)\alpha(y) + \beta(x)D(y) \quad (x, y \in R).$$

In addition, if there exists $x_0 \in R$ such that $d(x) = \beta(x)x_0 - x_0\alpha(x)$ holds for each $x \in R$, then d is called an inner (α, β)- derivation.
Note that if $R \subseteq S$ similarly the derivation and (α, β)-derivation $D : R \longrightarrow S$ can be defined (for further results see [5],[6],[7]).

Definition 2.1 A mapping σ from a ring R into a normed linear space S is approximately additive if there is $\delta > 0$ such that

$$||\sigma(x+y) - \sigma(x) - \sigma(y)|| \leqslant \delta \quad (x, y \in R).$$

And is approximately multiplicative if there is $\varepsilon > 0$ such that

$$||\sigma(xy) - \sigma(x)\sigma(y)|| \leqslant \varepsilon \quad (x, y \in R).$$

Definition 2.2 A mapping D from a normed linear space R into R is an approximate multiplicative derivation if there is $\delta > 0$ such that

$$||D(xy) - D(x)y - xD(y)|| \leqslant \delta \quad (x, y \in R).$$

And is an approximate multiplicative (α, β)-derivation if there is $\varepsilon > 0$ such that

$$||D(xy) - D(x)\alpha(y) - \beta(x)D(y)|| \leqslant \varepsilon \quad (x, y \in R).$$

Definition 2.3 Let R be a ring and S be a normed space. If for given $\epsilon > 0$ and for an approximate additive mapping $f : R \to S$ there exists a unique additive mapping $g : R \to S$ such that $\| f(x) - g(x) \| \leqslant \epsilon$ then we say the functional equation for additive functions is stable. In a situation where an approximate additive mapping must be a true additive mapping we say that the equation of the additive mapping is superstable.
In a similar fashion we can define stability and superstability of the functional equations of multiplicative functions and multiplicative derivations and multiplicative (α, β)-derivations.

Definition 2.4 A ring R is called a prime ring if $xRy = 0$ for $x, y \in R$, implies that $x = 0$ or $y = 0$.

Let X be a Banach space over $\mathbb{F} \in \{\mathbb{R}, \mathbb{C}\}$. We denote by $\mathcal{B}(X)$, the algebra of all bounded linear operators of X and $\mathcal{F}(X)$ the subalgebra of all bounded linear finite rank operators and $\mathcal{F}_1(X)$ the subalgebra of all bounded linear rank one operators. We shall call a subalgebra $\mathcal{A}$ of $\mathcal{B}(X)$ standard provided $\mathcal{A}$ contains $\mathcal{F}(X)$.

Definition 2.5 Let $\mathcal{A}$ be a standard operator algebra on a Banach space X.
A mapping $D : \mathcal{A} \longrightarrow \mathcal{B}(X)$ is called a linear derivation if
(i) $D(\lambda A) = \lambda D(A)$.
(ii) $D(A + B) = D(A) + D(B)$.
(iii) $D(AB) = AD(B) + D(A)B$.
For each $A, B \in \mathcal{A}$ and $\lambda \in \mathbb{F}$.
A mapping satisfying (ii) and (iii) is called a ring derivation. Multiplicative derivations are mapping satisfying only (iii). Linear (α, β)-derivation and ring (α, β)-derivation are defined similarly.
Given a Banach algebra A it is also to consider $n \times n$ matrix algebra $M_n(A)$ with the following standard operations,
$(a_{ij}) + (b_{ij}) = (a_{ij} + b_{ij})$, $\lambda(a_{ij}) = (\lambda a_{ij})$, $(a_{ij})(b_{ij}) = (\sum_{k=1}^{n} a_{ik}b_{kj})$ $(i, j = 1, 2, ..., n)$
And the norm $||(a_{ij})|| = sup_{1 \leqslant i \leqslant n}(||a_{i1}|| + ... + ||a_{in}||)$.
Note that if X is finite dimensional vector space then all norms defined on X are equivalent. So if A is finite dimensional then the above norm on $M_n(A)$ is equivalent to the operator norm.

Let X be a Banach space and X^* the dual space of X . If $x \in X$ and $f \in X^*$, then $x \otimes f$ denotes the operator defined by $(x \otimes f)(z) = f(z)x \quad (z \in X)$.
In particular if H is a Hilbert space and $x, y \in H$, then $x \otimes y$ denotes the operator defined by $(x \otimes y)(z) = \langle z, y \rangle x \qquad (z \in H)$.
Clearly if $A \in B(X)$, then $A(x \otimes f) = A(x) \otimes f$.

Definition 2.6 A Banach space X is called simple if $\mathcal{B}(X)$ has a unique nontrivial norm-closed two- sided ideal. For example, $l^p \quad (1 \leqslant p < \infty), c_0$ (The Banach space of all sequences which converges to zero, with l^∞ norm) and a separable infinite dimensional Hilbert space H are simple. In this case, the norm closure of all the finite rank operators is the ideal of compact operators, which is dense in $\mathcal{B}(X)$ with weak operator topology and is the unique nontrivial norm-closed two-sided ideal of $\mathcal{B}(X)$. (for further results see [1],[11],[13]).

3. Main Results

Lemma 3.1 [10] Let R be a ring containing a family $\{e_\alpha : \alpha \in A\}$ of idempotent which satisfies:
(1) $xR = 0$ implies $x = 0$.
(2)if $e_\alpha Rx = 0$ for each $\alpha \in A$, then $x = 0$ (and hence $Rx = 0$ implies $x = 0$).
(3)for each $\alpha \in A$, $e_\alpha x e_\alpha R(1 - e_\alpha) = 0$ implies $e_\alpha x e_\alpha = 0$.
Then every multiplicative isomorphism σ of R onto a arbitrary ring is additive.

As a special case of Lemma 3.1, we conclude the following theorem:

Theorem 3.2 Suppose that $\mathcal{R}$ is a ring containing a family $\{e_\alpha\}_{\alpha\in A}$ of idempotents, such that for each $\alpha \in A$ and $x \in \mathcal{R}$ satisfies the following conditions:
(i) $x\mathcal{R} = 0$ implies $x = 0$;
(ii) $e_\alpha \mathcal{R} x = 0$ implies $x = 0$;
(iii) If $e_\alpha x e_\alpha R(1 - e_\alpha) = 0$ then $e_\alpha x e_\alpha = 0$.
If α and β are ring homomorphisms on $\mathcal{R}$ and at least one of α and β is one to one then every multiplicative $(\alpha, \beta) - derivation$ of $\mathcal{R}$ is additive.

Proof. Let $d : \mathcal{R} \to \mathcal{R}$ be a multiplicative $(\alpha, \beta) - derivation$, and let
$\mathcal{S} = \left\{ \begin{pmatrix} \beta(x) & d(x) \\ 0 & \alpha(x) \end{pmatrix} | x \in \mathcal{R} \right\}$. Obviously $\mathcal{S}$ is a ring. Define $\sigma : \mathcal{R} \to \mathcal{S}$ by
$\sigma(x) = \begin{pmatrix} \beta(x) & d(x) \\ 0 & \alpha(x) \end{pmatrix}$, for each $x \in \mathcal{R}$. Then σ is onto and one to one, since one of α and β is one to one.
For every $x, y \in \mathcal{R}$, we have

$$\begin{aligned}
\sigma(xy) &= \begin{pmatrix} \beta(xy) & d(xy) \\ 0 & \alpha(xy) \end{pmatrix} \\
&= \begin{pmatrix} \beta(x)\beta(y) & d(x)\alpha(y) + \beta(x)d(y) \\ 0 & \alpha(x)\alpha(y) \end{pmatrix} \\
&= \begin{pmatrix} \beta(x) & d(x) \\ 0 & \alpha(x) \end{pmatrix} \\
&= \begin{pmatrix} \beta(y) & d(y) \\ 0 & \alpha(y) \end{pmatrix} \\
&= \sigma(x)\sigma(y).
\end{aligned}$$

Then σ is multiplicative. Hence it is an isomorphism and by Lemma 3.1, it is additive.

$$\begin{aligned}
\sigma(x + y) &= \begin{pmatrix} \beta(x + b) & d(x + y) \\ 0 & \alpha(x + y) \end{pmatrix} \\
&= \sigma(x) + \sigma(y) \\
&= \begin{pmatrix} \beta(x) + \beta(y) & d(x) + d(y) \\ 0 & \alpha(x) + \alpha(y) \end{pmatrix}
\end{aligned}$$

Hence d is additive. ■

Lemma 3.3 [8] Let X be a complex Banach space, α and β be mappings from $\mathcal{B}(X)$

into itself. Let $D : \mathcal{B}(X) \to \mathcal{B}(X)$ be a linear (α, β)-derivation. Then D is continuous if one of the following conditions holds:
(i) α is an automorphism, β is continuous at 0 and the set $\{\beta(T) : T \in \mathcal{F}_1(X)\}$ seperates the points of X in the sense that, for each pair $\xi, \eta \in X$ with $\xi \neq \eta$, there is a rank one operator T such that $\beta(T)\xi \neq \beta(T)\eta$, equivalently, the set $\{\beta(T) : T \in \mathcal{F}_1(X)\}$ has no nonzero right annihilators in $\mathcal{B}(X)$.
(ii) β is an automorphism, α is continuous at 0 and the set $\{\alpha(T) : T \in \mathcal{F}_1(X)\}$ has no nonzero right annihilators in $\mathcal{B}(X)$.
(iii) α and β are continuous at 0, $span\{\alpha(T)\xi : T \in \mathcal{F}_1(X), \xi \in X\}$ is dense in X and there is a rank one S such that $\beta(S)$ is injective.
(iv) X is simple and α, β are surjective and continuous at zero.
(v) α, β are surjective and multiplicative and there are rank one operators T_0 and S_0 such that $\alpha(T_0) \neq 0$ and $\beta(S_0) \neq 0$.
Moreover, if either (i), or X is reflexive and (ii), holds, D is (α, β)-inner.

Let $\mathbb{R}_+$ the set of all nonnegative real numbers. We prove that the equation of a special multiplicative $(\alpha, \beta) - derivation$ is superstable on standard operator algebras.

Theorem 3.4 Let X be a complex Banach space with $dimX > 1$ and suppose that $\phi : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is a mapping such that $lim_{t\to\infty} \frac{\phi(t)}{t} = 0$ and $D : \mathcal{B}(X) \longrightarrow \mathcal{B}(X)$ is a mapping satisfying $D(\lambda A) = \lambda D(A) \qquad (\lambda \in \mathbb{C}, A \in \mathcal{B}(X))$ and

$$||D(AB) - \beta(A)D(B) - D(A)\alpha(B)|| < \phi(||A||||B||) \quad (A, B \in \mathcal{B}(X)).$$

If α, β are ring homomorphisms on $\mathcal{B}(X)$ and at least one of α and β is one to one in which α is a scalar multiplicative preservinng map then the following statements holds:
(1) D is (α, β)-inner if either condition (i) or when X is reflexive, condition (ii) of lemma 3.3 holds.
(2) D is continuous if one of conditions (i),(ii), (iii), (iv) and (v) of Lemma 3.3 holds.

Proof. At first we show that if $A, B \in \mathcal{B}(X)$, then $A\mathcal{B}(X)B = 0$ implies $A = 0$ or $B = 0$. In fact if $B \neq 0$, then there exists $z \in A$ such that $B(z) \neq 0$ and then from Hahn Banach theorem there exists $f \in X^*$ such that $f(B(z)) \neq 0$. Now for every arbitrary $x \in X$ we have:

$$A\mathcal{B}(X)B = 0$$

$$A(x \otimes f)B(z) = 0$$

$$(Ax \otimes f)(B(z)) = 0$$

$$f(B(z))A(x) = 0$$

$$A(x) = 0 \qquad\qquad (x \in X)$$

$$A = 0.$$

Furthermore if $\{x_i\}$ is a base for X and j be considered fixed we can define $T : X \to X$ by $T(\sum a_i x_i) = a_j x_j$, then clearly T is a nontrivial idempotent of rank one.
Therefore $\mathcal{B}(X)$ is a prime ring with nontrivial idempotent and so satisfies the conditions of Theorem 3.2 and so every multiplicative (α, β)- derivation on $\mathcal{B}(X)$ is a ring (α, β)-derivation.
Replacing B by tB in which t is a positive real number in $||D(AB) - \beta(A)D(B) - D(A)\alpha(B)|| < \phi(||A||||B||)$, and then by dividing the above inequality by t, we obtain $||D(AB) - \beta(A)D(B) - D(A)\alpha(B)|| < \frac{\phi(t||A||||B||)}{t}$. By taking a limit when $t \to \infty$ we see that D is multiplicative (α, β)- derivation and hence the above observations and assumption imply that D is a linear (α, β)-derivation. Now, the result follows from Lemma 3.3. ■

Now we want extend Theorem 3.4 for the case D is not nessecairly scalar multiplicative preserving maps.

Theorem 3.5 Let X be a Banach space with $dimX > 1$ and $\mathcal{A}$ be a standard operator algebra on X. Assume that $\phi : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is a function satisfying $lim_{t \to \infty} \frac{\phi(t)}{t} = 0$. Suppose $\alpha : \mathcal{A} \longrightarrow \mathcal{A}$ is an algebra automorphism and $\beta : \mathcal{A} \longrightarrow \mathcal{A}$ be a ring automorphism and suppose that $D : \mathcal{A} \longrightarrow \mathcal{B}(X)$ is a mapping such that
$||D(AB) - \beta(A)D(B) - D(A)\alpha(B)|| < \phi(||A||||B||)$ $(A, B \in \mathcal{A})$.
Then D is multiplicative $(\alpha, \beta)-$ derivation.

Proof. Let us define a mapping $\phi : \mathcal{A} \longrightarrow \mathcal{B}(X \oplus X)$ by

$$\phi(A) = \begin{pmatrix} \beta(A) & D(A) \\ 0 & \alpha(A) \end{pmatrix}$$

We have

$$\phi(AB) - \phi(A)\phi(B)$$

$$= \begin{pmatrix} \beta(AB) & D(AB) \\ 0 & \alpha(AB) \end{pmatrix} - \begin{pmatrix} \beta(A) & D(A) \\ 0 & \alpha(A) \end{pmatrix} \begin{pmatrix} \beta(B) & D(B) \\ 0 & \alpha(B) \end{pmatrix}$$

$$= \begin{pmatrix} \beta(AB) & D(AB) \\ 0 & \alpha(AB) \end{pmatrix} - \begin{pmatrix} \beta(A)\beta(B) & \beta(A)D(B) + D(A)\alpha(B) \\ 0 & \alpha(A)\alpha(B) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & D(AB) - \beta(A)D(B) - D(A)\alpha(B) \\ 0 & 0 \end{pmatrix},$$

Consequently
$$||\phi(AB) - \phi(A)\phi(B)|| = \left|\left| \begin{pmatrix} 0 & D(AB) - \beta(A)D(B) - D(A)\alpha(B) \\ 0 & 0 \end{pmatrix} \right|\right|$$

$=Sup(||0|| + ||D(AB) - \beta(A)D(B) - D(A)\alpha(B)||, ||0|| + ||0||)$

$=||D(AB) - \beta(A)D(B) - D(A)\alpha(B)|| < \phi(||A||||B||) \qquad (A, B \in \mathcal{A}).$

So we have
$(\phi(AB) - \phi(A)\phi(B))\ \phi(C)$

$$= \begin{pmatrix} 0 & D(AB) - \beta(A)D(B) - D(A)\alpha(B) \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \beta(C) & D(C) \\ 0 & \alpha(C) \end{pmatrix}$$
$$= \begin{pmatrix} 0 & (D(AB) - \beta(A)D(B) - D(A)\alpha(B))\alpha(C) \\ 0 & 0 \end{pmatrix}.$$

It follows that for arbitrary $A, B, C \in \mathcal{A}$, we have

$||(D(AB) - \beta(A)D(B) - D(A)\alpha(B))\alpha(C)||$

$= \left|\left| \begin{pmatrix} 0 & (D(AB) - \beta(A)D(B) - D(A)\alpha(B))\alpha(C) \\ 0 & 0 \end{pmatrix} \right|\right|$

$= \|(\phi(AB) - \phi(A)\phi(B))\phi(C)\|$

$= ||\phi(AB)\phi(C) - \phi(A)\phi(BC) + \phi(A)\phi(BC) - \phi(A)\phi(B)\phi(C)||$

$\leqslant ||\phi(AB)\phi(C) - \phi(A)\phi(BC)|| + ||\phi(A)\phi(BC) - \phi(A)\phi(B)\phi(C)||$

$= ||\phi(AB)\phi(C) - \phi(ABC) + \phi(ABC) - \phi(A)\phi(BC)|| + ||\phi(A)\phi(BC) - \phi(A)\phi(B)\phi(C)||$

$\leqslant ||\phi(AB)\phi(C) - \phi(ABC)|| + ||\phi(ABC) - \phi(A)\phi(BC)|| + ||\phi(A)||||\phi(BC) - \phi(B)\phi(C)||$

$< \phi(||AB||||C||) + \phi(||A||||BC||) + ||\phi(A)||\phi(||B||||C||).$

Replacing C by tC in which t is a positive real number;

$||(D(AB) - \beta(A)D(B) - D(A)\alpha(B))\alpha(tC)||$

$< \phi(t||AB||||C||) + \phi(t||A||||BC||) + ||\phi(A)||||\phi(t||B||||C||)||,$

Dividing the above inequality by t, we get

$0 \leqslant ||(D(AB) - \beta(A)D(B) - D(A)\alpha(B))\alpha(C)|| < \frac{\phi(t||AB||||C||)}{t} + \frac{\phi(t||A||||BC||)}{t} + ||\phi(A)||\frac{\phi(t||B||||C||)}{t},$
Taking t to infinity we have

$$||(D(AB) - \beta(A)D(B) - D(A)\alpha(B))\alpha(C)|| = 0.$$

$$(D(AB) - \beta(A)D(B) - D(A)\alpha(B))\alpha(C) = 0.$$

For any arbitrary $E \in \mathcal{A}$, there exists $C \in \mathcal{A}$, such that $\alpha(C) = E$, since α is onto. Hence

for any arbitrary $E \in \mathcal{A} \supseteq \mathcal{F}(X)$, we have $(D(AB) - D(A)\alpha(B) - \beta(A)D(B))E = 0$. Therefore $(D(AB) - D(A)\alpha(B) - \beta(A)D(B))F(X) = 0$.
So with the similar argument in Theorem 3.4 with application Hahn Banach theorem we have:

$$D(AB) - D(A)\alpha(B) - \beta(A)D(B) = 0.$$

$$D(AB) = D(A)\alpha(B) + \beta(A)D(B).$$

as desired. ∎

Corollary 3.6 Let X and $\mathcal{A}$ and ϕ be the same as in Theorem 3.5 and $D : \mathcal{A} \to \mathcal{B}(X)$ be a mapping satisfies

$$\| D(AB) - AD(B) - D(A)B \| < \phi(\| A \| \| B \|) \quad (A, B \in \mathcal{A})$$

then D is a multiplicatie derivation.

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