Journal of
Linear and Topological Algebra
Vol. 04, *No.* 02, 2015, 115- 129

Second order linear differential equations with generalized trapezoidal intuitionistic Fuzzy boundary value

S. P. Mondal[a*], T. K. Roy[b]

[a] *Department of Mathematics, National Institute of Technology, Agartala, Jirania-799046, Tripura, India;*
[b] *Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah-711103, West Bengal, India.*

Received 19 June 2015; Revised 2 August 2015; Accepted 17 September 2015.

Abstract. In this paper the solution of a second order linear differential equations with intuitionistic fuzzy boundary value is described. It is discussed for two different cases: coefficient is positive crisp number and coefficient is negative crisp number. Here fuzzy numbers are taken as generalized trapezoidal intutionistic fuzzy numbers (GTrIFNs). Further a numerical example is illustrated.

Keywords: Fuzzy set, fuzzy differential equation, generalized trapezoidal intutionistic fuzzy number.

2010 AMS Subject Classification: Primary: 34A07, Secondary: 03E72.

1. Introduction

1.1 *Fuzzy and intutionistic fuzzy set theory*

Zadeh [1] and Dubois and Parade [2] were the first who introduced the conception based on fuzzy number and fuzzy arithmetic. The generalizations of fuzzy sets theory [1] is considered as Intuitionistic fuzzy set (IFS). Out of several higher-order fuzzy sets, IFS was first introduced by Atanassov [3] and have been found to be suitable to deal with unexplored areas.The fuzzy set considers only the degree of belongingness and non belongingness. Fuzzy set theory does not incorporate the degree of hesitation (i.e.,degree

*Corresponding author.
E-mail address: sankar.res07@gmail.com (S. P. Mondal).

Print ISSN: 2252-0201
Online ISSN: 2345-5934

of non-determinacy defined as, 1- sum of membership function and non-membership function).To handle such situations, Atanassov [4] explored the concept of fuzzy set theory by intuitionistic fuzzy set (IFS) theory.The degree of acceptance is only considered in Fuzzy Sets but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one [4].

1.2 *Fuzzy derivative and fuzzy differential equation*

The topic "fuzzy differential equation"(FDE) has been rapidly developing in recent years. The appliance of fuzzy differential equations is a inherent way to model dynamic systems under possibilistic uncertainty [5]. The concept of the fuzzy derivative was first introduced by Chang and Zadeh [6] and it was followed up by Dubois and Prade [7]. Other methods have been discussed by Puri and Ralescu [8] and Goetschel and Voxman [9]. The concept of differential equations in a fuzzy environment was first formulated by Kaleva [10]. In fuzzy differential equation all derivative is deliberated as either Hukuhara or generalized derivatives. The Hukuhara differentiability has a imperfection (see [12], [19]). The solution turns fuzzier as time goes by. Bede exhibited that a large class of BVPs has no solution if the Hukuhara derivative is used [11]. To overcome this difficulty, the concept of a generalized derivative was developed [[12], [13]) and fuzzy differential equations were discussed using this concept (see [14], [15], [16], [17]). Khastan and Nieto found solutions for a large enough class of boundary value problems using the generalized derivative [18].Bede in [26] discussed the generalized differentiability for fuzzy valued functions. Pointedly the disadvantage of strongly generalized differentiability of a function in comparison H-differentiability is that, a fuzzy differential equation has no unique solution [20]. Recently, Stefanini and Bede by the concept of generalization of the Hukuhara difference for compact convex set [21], introduced generalized Hukuhara differentiability [22] for fuzzy valued function and they demonstrated that, this concept of differentiability have relationships with weakly generalized differentiability and strongly generalized differentiability. Recently Gasilov et. all. [25] solve the fuzzy initial value problem by a new technique where Barros et. all. [33] solve fuzzy differential equation via fuzzification of the derivative operator. Recently the research on fuzzy boundary value is grew attention among all. Armand and Gouyandeh [23] solve two point fuzzy boundary value problems using variational iteration method. Gasilov et. all. [24] solve linear differential equation with fuzzy boundary value. Lopez [27] find the existence of solutions to periodic boundary value problems for linear differential equation.

1.3 *Intutionistic fuzzy differential equation*

Intutionistic FDE is very rare. Melliani and Chadli [28] solve partial differential equation with intutionistic fuzzy number. Abbasbandy and Allahviranloo [29] discussed numerical Solution of fuzzy differential equations by runge-kutta method and in the intuitionistic fuzzy environment.. Lata and Kumar [30] worked on time-dependent intuitionistic fuzzy differential equation and its application to analyze the intutionistic fuzzy reliability of industrial system. First order homogeneous ordinary differential equation with initial value as triangular intuitionistic fuzzy number is described by Mondal and Roy [31]. System of differential equation with initial value as triangular intuitionistic fuzzy number and its application is solved by Mondal and Roy [32].

1.4 *Novelties*

In spite of above mentioned developments, following lacunas are still exists in the formulation and solution of the fuzzy boundary value problem, which are summarized below:

(i) Though there are some articles of fuzzy boundary value problem was solved but till now none has solve boundary value problem with intuitionistic fuzzy number.
(ii) Here also second order differential equation is solved with intuitionistic fuzzy number.
(iii) The intutionistic fuzzy number is also taken as generalized trapezoidal intuitionistic fuzzy number.

1.5 *Structure of the paper*

The paper is organized as follows. In Section 2, the basic concept on fuzzy number and fuzzy derivative are discussed. Also basic definition and properties of generalized trapezoidal intuitionistic fuzzy number are defined. In Section 3 we solve the intutionistic fuzzy boundary value problem. In Section 4 the proposed method is illustrated by an example. The conclusion and future research scope is drawn in the last section 5.

2. Preliminary concept

Definition 2.1:Fuzzy Set: A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \epsilon A, \mu_{\tilde{A}}(x) \epsilon [0,1]\}$. In the pair $(x, \mu_{\tilde{A}}(x))$ the first element belongs to the classical set A, the second element $\mu_{\tilde{A}}(x)$, belongs to the interval $[0,1]$, called membership function.

Definition 2.2:α-cut of a fuzzy set: The α-level set (or, interval of confidence at level α or α-cut) of the fuzzy set $\tilde{A}$ of X that have membership values in A greater than or equal to α i.e., $\tilde{A} = \{x : \mu_{\tilde{A}}(x) \geqslant \alpha, x \epsilon X, \alpha \epsilon [0,1]\}$.

Definition 2.3: Fuzzy number:A fuzzy number is an extension of a regular number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1.This weight is called membership function. Thus a fuzzy number is a convex and normal fuzzy set.

Definition 2.4:Intuitionistic Fuzzy set: Let a set X be fixed. An IFS $\tilde{A}^i$ in X is an object having the form $\tilde{A}^i = \{< x, \mu_{\tilde{A}^i}(x), \vartheta_{\tilde{A}^i}(x) > 0 : x \epsilon X\}$, where the $\mu_{\tilde{A}^i}(x) : X \to [0,1]$ and $\vartheta_{\tilde{A}^i}(x) : X \to [0,1]$ define the degree of membership and degree of non-membership respectively, of the element $x \epsilon X$ to the set $\tilde{A}^i$, which is a subset of X, for every element of $x \epsilon X, 0 < \mu_{\tilde{A}^i}(x) + \vartheta_{\tilde{A}^i}(x) \leqslant 1$.

Definition 2.5:Intuitionistic Fuzzy number: An IFN $\tilde{A}^i$ is defined as follows
(i) an intuitionistic fuzzy subject of real line.
(ii) normal. i.e., there is any $x_0 \epsilon R$ such that $\mu_{\tilde{A}^i}(x_0) = 1$(so $\vartheta_{\tilde{A}^i}(x_0) = 0$)
(iii) a convex set for the membership function $\mu_{\tilde{A}^i}(x)$, i.e.,
$\mu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \geqslant min(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)) \forall x_1, x_2 \epsilon R, \lambda \epsilon [0,1]$
(iv)a concave set for the non-membership function $\vartheta_{\tilde{A}^i}(x)$, i.e.,
$\vartheta_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \leqslant min(\vartheta_{\tilde{A}^i}(x_1), \vartheta_{\tilde{A}^i}(x_2)) \forall x_1, x_2 \epsilon R, \lambda \epsilon [0,1]$

Definition 2.6: Trapezoidal Intuitionistic Fuzzy number: A TrIFN $\tilde{A}^i$ is a subset of IFN in R with following membership function and non membership function as follows:

$$\mu_{\tilde{A}_i}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leqslant x < a_2 \\ 1 & \text{if } a_2 \leqslant x \leqslant a_3 \\ \frac{a_4-x}{a_4-a_3} & \text{if } a_3 < x \leqslant a_4 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\vartheta_{\tilde{A}_i}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'} & \text{if } a_1' \leqslant x < a_2 \\ 0 & \text{if } a_2 \leqslant x \leqslant a_3 \\ \frac{x-a_3}{a_4'-a_3} & \text{if } a_3 < x \leqslant a_4' \\ 1 & \text{otherwise} \end{cases}$$

where $a_1' \leqslant a_2 \leqslant a_3 \leqslant a_4'$, $a_1 \leqslant a_2 \leqslant a_3 \leqslant a_4$ and TrIFN is denoted by $\tilde{A}_{TrIFN} = (a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4')$

Note 2.1: Here $\mu_{\tilde{A}_i}(x)$ increases with constant rate for $x\epsilon[a_1, a_2]$ and decreases with constant rate for $x\epsilon[a_3, a_4]$ but $\vartheta_{\tilde{A}_i}(x)$ decreases with constant rate for $x\epsilon[a_1', a_2]$ and increases with constant rate for $x\epsilon[a_3, a_4']$

Definition 2.7: Generalized Intuitionistic Fuzzy Number: An IFN $\tilde{A}^i$ is defined as follows

(i) an intuitionistic fuzzy subject of real line.

(ii) normal. i.e., there is any $x_0 \epsilon R$ such that $\mu_{\tilde{A}^i}(x_0) = \omega$(so $\vartheta_{\tilde{A}^i}(x_0) = \sigma$) for $0 < \omega + \sigma \leqslant 1$.

(iii) a convex set for the membership function $\mu_{\tilde{A}^i}(x)$, i.e.,
$\mu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \geqslant min(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)) \forall x_1, x_2 \epsilon R, \lambda \epsilon [0, \omega]$

(iv)a concave set for the non-membership function $\vartheta_{\tilde{A}^i}(x)$, i.e.,
$\vartheta_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \leqslant min(\vartheta_{\tilde{A}^i}(x_1), \vartheta_{\tilde{A}^i}(x_2)) \forall x_1, x_2 \epsilon R, \lambda \epsilon [\sigma, 1]$

(v) $\mu_{\tilde{A}^i}$ and $\vartheta_{\tilde{A}^i}$ is continuous mapping from R to the closed interval $[0, \omega]$ and $[\sigma, 1]$ respectively and $x_0 \epsilon R$, the relation $0 \leqslant \mu_{\tilde{A}^i} + \vartheta_{\tilde{A}^i} \leqslant 1$ holds.

Definition 2.8: Generalized Trapezoidal Intuitionistic Fuzzy number: A GTrIFN $\tilde{A}^i$ is a subset of IFN in R with following membership function and non membership function as follows:

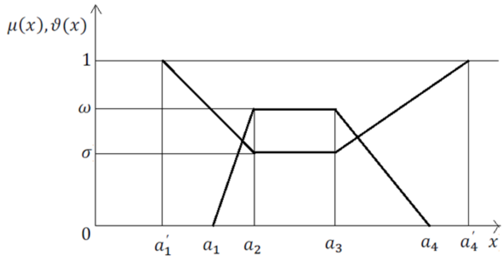

Figure 1. Generalized trapezoidal intutionistic fuzzy number

$$\mu_{\tilde{A}_i}(x) = \begin{cases} \omega\frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leqslant x < a_2 \\ \omega & \text{if } a_2 \leqslant x \leqslant a_3 \\ \omega\frac{a_4-x}{a_3-a_3} & \text{if } a_3 < x \leqslant a_4 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\vartheta_{\tilde{A}_i}(x) = \begin{cases} \sigma\frac{a_2-x}{a_2-a_1'} & \text{if } a_1' \leqslant x < a_2 \\ \sigma & \text{if } a_2 \leqslant x \leqslant a_3 \\ \sigma\frac{x-a_3}{a_4'-a_3} & \text{if } a_3 < x \leqslant a_4' \\ 0 & \text{otherwise} \end{cases}$$

where $a_1' \leqslant a_2 \leqslant a_3 \leqslant a_4'$, $a_1 \leqslant a_2 \leqslant a_3 \leqslant a_4$ and GTrIFN is denoted by $\tilde{A}_{GTrIFN} = ((a_1, a_2, a_3, a_4; \omega), (a_1', a_2, a_3, a_4'; \sigma))$

Definition 2.9:Non negative GTrIFN: A GTrIFN $\widetilde{A}^i_{GTrIFN} = ((a_1, a_2, a_3, a_4; \omega), (a_1', a_2, a_3, a_4'; \sigma))$ iff $a_1' \geqslant 0$.

Definition 2.10:Equality of two GTrIFN: A GTrIFN $\widetilde{A}^i_{GTIFN} = ((a_1, a_2, a_3, a_4; \omega_1), (a_1', a_2, a_3, a_4'; \sigma_1))$ and $\widetilde{B}^i_{GTIFN} = ((b_1, b_2, b_3, b_4; \omega_2), (b_1', b_2, b_4, b_4'; \sigma_2))$ are said to be equal iff $a_1 = b_1$,$a_2 = b_2$,$a_3 = b_3$,$a_4 = b_4$,$a_1' = b_1'$,$a_4' = b_4'$,$\omega_1 = \omega_2$ and $\sigma_1 = \sigma_2$.

Definition 2.11: α-cut set: α-cut set of a GTrIFN $\widetilde{A}^i_{GTrIFN} = ((a_1, a_2, a_3, a_4; \omega), (a_1', a_2, a_3, a_4'; \sigma))$ is a crisp subset of R which is defined as follows

$A_\alpha = \{x : \mu_{\tilde{A}_i}(x) \geqslant \alpha\} = [A_1(\alpha), A_2(\alpha)] = [a_1 + \frac{\alpha}{\omega}(a_2 - a_1), a_4 - \frac{\alpha}{\omega}(a_4 - a_3)]$

Definition 2.12: β-cut set: β-cut set of a GTrIFN $\widetilde{A}^i_{GTIFN} = ((a_1, a_2, a_3, a_4; \omega), (a_1', a_2, a_3, a_4'; \sigma))$ is a crisp subset of R which is defined as follows

$A_\beta = \{x : \vartheta_{\tilde{A}_i}(x) \leqslant \beta\} = [A_1'(\beta), A_2'(\beta)] = [a_2 - \beta(a_2 - a_1'), a_3 + \beta(a_4' - a_3)]$

Definition 2.13: (α,β)-cut set: (α,β)-cut set of a GTrIFN $\widetilde{A}^i_{GTrIFN} = ((a_1,a_2,a_3,a_4;\omega),(a_1^{'},a_2,a_3,a_4^{'};\sigma))$ is a crisp subset of R which is defined as follows
$A_{\alpha,\beta} = \{[A_1(\alpha),A_2(\alpha)];[A_1^{'}(\beta),A_2^{'}(\beta)]\}, 0<\alpha+\beta\leqslant\omega,\sigma,\alpha\epsilon[0,\omega],\beta\epsilon[\sigma,1]$

Definition 2.14: Addition of two GTrIFN: Let two $\widetilde{A}^i_{GTrIFN} = ((a_1,a_2,a_3,a_4;\omega_1),(a_1^{'},a_2,a_3,a_4^{'};\sigma_1))$ and $\widetilde{B}^i_{GTrIFN} = ((b_1,b_2,b_3,b_4;\omega_2),(b_1^{'},b_2,b_3,b_4^{'};\sigma_2))$ be GTrIFN, then the addition of two GTrIFN is given by

$\widetilde{A}^i_{GTrIFN}\bigoplus\widetilde{B}^i_{GTrIFN} = ((a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4;\omega),(a_1^{'}+b_1^{'},a_2+b_2,a_3+b_3,a_4^{'}+b_4^{'};\sigma))$
where $\omega = min\{\omega_1,\omega_2\}$ and $\sigma_1 = min\{\sigma_1,\sigma_2\}$.

Definition 2.15: Subtraction of two GTrIFN: Let two $\widetilde{A}^i_{GTrIFN} = ((a_1,a_2,a_3m,a_4;\omega_1),(a_1^{'},a_2,a_3,a_3^{'};\sigma_1))$ and
$\widetilde{B}^i_{GTrIFN} = ((b_1,b_2,b_3,b_4;\omega_2),(b_1^{'},b_2,b_3,b_4^{'};\sigma_2))$ be GTrIFN, then the subtraction of two GTrIFN is given by

$\widetilde{A}^i_{GTrIFN}\ominus\widetilde{B}^i_{GTrIFN} = ((a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1;\omega),(a_1^{'}-b_4^{'},a_2-b_3,a_3-b_2,a_4^{'}-b_1^{'};\sigma))$
where $\omega = \min\{\omega_1,\omega_2\}$ and $\sigma_1 = \min\{\sigma_1,\sigma_2\}$.

Definition 2.16: Multiplication by a scalar: Let $\widetilde{A}^i_{GTrIFN} = ((a_1,a_2,a_3;\omega),(a_1^{'},a_2,a_3^{'};\sigma))$ and k is a scalar then $k\widetilde{A}^i_{GTrIFN}$ is also a GTrIFN and is defined as

$$k\widetilde{A}^i_{GTrIFN} = \begin{cases} ((ka_1,ka_2,ka_3;\omega),(ka_1^{'},ka_2,ka_3,ka_4^{'};\sigma)) \text{ if } k>0 \\ ((ka_3,ka_2,ka_1;\omega),(ka_4^{'},ka_3,ka_2,ka_1^{'};\sigma)) \text{ if } k<0 \end{cases}$$

where $0<\omega,\sigma\leqslant 1$.

Definition 2.17: Multiplication of two GTIFN: Let two $\widetilde{A}^i_{GTrIFN} = ((a_1,a_2,a_3,a_4;\omega_1),(a_1^{'},a_2,a_3,a_4^{'};\sigma_1))$ and $\widetilde{B}^i_{GTrIFN} = ((b_1,b_2,b_3,b_4;\omega_2),(b_1^{'},b_2,b_3,b_3^{'};\sigma_2))$ be GTrIFN, then the multiplication of two GTIFN is given by
$\widetilde{A}^i_{GTrIFN}\bigotimes\widetilde{B}^i_{GTrIFN} = ((a_1b_1,a_2b_2,a_3b_3,a_4b_4;\omega),(a_1^{'}b_1^{'},a_2b_2,a_3b_3,a_4^{'}b_4^{'};\sigma))$
where $\omega = \min\{\omega_1,\omega_2\}$ and $\sigma_1 = \min\{\sigma_1,\sigma_2\}$.

Definition 2.18: Division of two GTrIFN: Let two $\widetilde{A}^i_{GTrIFN} = ((a_1,a_2,a_3,a_4;\omega_1),(a_1^{'},a_2,a_3,a_4^{'};\sigma_1))$ and $\widetilde{B}^i_{GTrIFN} = ((b_1,b_2,b_3,b_4;\omega_2),(b_1^{'},b_2,b_3,b_4^{'};\sigma_2))$ be GTrIFN, then the multiplication of two GTIFN is given by
$\widetilde{A}^i_{GTrIFN}/\widetilde{B}^i_{GTrIFN} = ((\frac{a_1}{b_4},\frac{a_2}{b_3},\frac{a_3}{b_2},\frac{a_4}{b_1};\omega),(\frac{a_1^{'}}{b_4^{'}},\frac{a_2}{b_3},\frac{a_3}{b_2},\frac{a_4^{'}}{b_1^{'}};\sigma))$
where $\omega = \min\{\omega_1,\omega_2\}$ and $\sigma_1 = \min\{\sigma_1,\sigma_2\}$.

Theorem 2.1: [34] If ω and σ represent the maximum degree of membership and minimum degree of non-membership function respectively then they satisfy the conditions $0\leqslant\omega\leqslant 1$, $0\leqslant\sigma\leqslant 1$ and $0<\omega+\sigma\leqslant 1$.

Definition 2.19: Generalized Hukuhara difference: [20] The generalized Hukuhara difference of two fuzzy number $u,v\in\Re_F$ is defines as follows

$u \ominus_{gH} v = w$ is equivalent to $\begin{cases} (i) u = v \oplus w \\ (ii) v = u \oplus (-1)w \end{cases}$

Consider $[w]_\alpha = [w_1(\alpha), w_2(\alpha)]$, then $w_1(\alpha) = min\{u_1(\alpha) - v_1(\alpha), u_2(\alpha) - v_2(\alpha)\}$ and $w_2(\alpha) = max\{u_1(\alpha) - v_1(\alpha), u_2(\alpha) - v_2(\alpha)\}$

Here the parametric representation of a fuzzy valued function $f : [a, b] \to \Re_F$ is expressed by $[f(t)]_\alpha = [f_1(t, \alpha), f_2(t, \alpha)]$, $t \in [a, b], \alpha \in [0, 1]$.

Definition 2.20: Generalized Hukuhara derivative for first order: [20] The generalized Hukuhara derivative of a fuzzy valued function $f : (a, b) \to \Re_F$ at t_0 is defined as $f'(t_0) = lim_{h \to 0} \frac{f(t_0+h) \ominus_{gH} f(t_0)}{h}$

If $f'(t_0) \in \Re_F$ satisfying (2.1) exists, we say that f is generalized Hukuhara differentiable at t_0.

Also we say that $f(t)$ is (i)-gH differentiable at t_0 if

$$[f'(t_0)]_\alpha = [f_1'(t_0, \alpha), f_2'(t_0, \alpha)]$$

and $f(t)$ is (ii)-gH differentiable at t_0 if

$$[f'(t_0)]_\alpha = [f_2'(t_0, \alpha), f_1'(t_0, \alpha)]$$

Definition 2.21: Generalized Hukuhara derivative for second order: [23] The second order generalized Hukuhara derivative of a fuzzy valued function $f : (a, b) \to \Re_F$ at t_0 is defined as

$$f''(t_0) = lim_{h \to 0} \frac{f'(t_0+h) \ominus_{gH} f'(t_0)}{h}$$

If $f'' \in \Re_F$, we say that $f'(t_0)$ is generalized Hukuhara at t_0.

Also we say that $f'(t_0)$ is (i)-gH differentiable at t_0 if

$$f''(t_0, \alpha) = \begin{cases} (f_1'(t_0, \alpha), f_2'(t_0, \alpha)) \text{ if f be (i)-gH differentiable on (a,b)} \\ (f_2'(t_0, \alpha), f_1'(t_0, \alpha)) \text{ if f be (ii)-gH differentiable on (a,b)} \end{cases}$$

for all $\alpha \in [0, 1]$, and that $f'(t_0)$ is (ii)-gH differentiable at t_0 if

$$f''(t_0, \alpha) = \begin{cases} (f_2'(t_0, \alpha), f_1'(t_0, \alpha)) \text{ if f be (i)-gH differentiable on (a,b)} \\ (f_1'(t_0, \alpha), f_2'(t_0, \alpha)) \text{ if f be (ii)-gH differentiable on (a,b)} \end{cases}$$

for all $\alpha \in [0, 1]$.

Definition 2.22: Strong and week solution: If the solution of intutionistic fuzzy differential equation is of the form $[x_1(t, \alpha), x_2(t, \alpha); x_1'(t, \beta), x_2'(t, \beta)]$, the solution is called strong solution when

(i) $\frac{dx_1(t,\alpha)}{d\alpha} > 0$, $\frac{dx_2(t,\alpha)}{d\alpha} < 0$ $\forall \alpha \in [0, \omega], x_1(t, \omega) \leqslant x_2(t, \omega)$

(ii) $\frac{dx_1'(t,\beta)}{d\beta} < 0$, $\frac{dx_2(t,\beta)}{d\beta} > 0$ $\forall \beta \in [\sigma, 1], x_1'(t, \sigma) \leqslant x_2'(t, \sigma)$

Definition 2.23: (α, β)-cuts to Intutionistic fuzzy number: Let $[A_1(\alpha), A_2(\alpha); A_1'(\beta), A_2'(\beta)]$ be the (α, β)-cuts of a trapezoidal intutionistic fuzzy number $\widetilde{A}$ and ω, σ be the gradation of membership and non membership function respectively then the intutionistic fuzzy number is given by

$\widetilde{A} = ((A_1(\alpha = 0), A_1(\alpha = \omega), A_2(\alpha = \omega), A_2(\alpha = 0); \omega); (A_1'(\beta = \sigma), A_1'(\beta = 0), A_2'(\beta = 0), A_2'(\beta = \sigma); \sigma))$

3. Second order intutionistic fuzzy boundary value problem

Consider the differential equation $\frac{d^2x(t)}{dt^2} = kx(t)$, with boundary condition $x(0) = \widetilde{a}$ and $x(L) = \widetilde{b}$. Where $\widetilde{a}, \widetilde{b}$ are generalized trapezoidal intutionistic fuzzy number.

Let $\widetilde{a} = ((a_1, a_2, a_3, a_4; \omega_1), (a_1', a_2, a_3, a_4'; \sigma_1))$ and $\widetilde{b} = ((b_1, b_2, b_3, b_4; \omega_2), (b_1', b_2, b_3, b_4'; \sigma_2))$

3.1 *Second order intutionistic fuzzy boundary value problem with coefficients is positive constant i.e., $k > 0$*

Here two cases arise

Case 3.1.1: When $x(t)$ is (i)-gH differentiable and $\frac{dx(t)}{dt}$ is (i)-gH then we have

$\frac{d^2x_1(t,\alpha)}{dt^2} = kx_1(t, \alpha)$

$\frac{d^2x_2(t,\alpha)}{dt^2} = kx_2(t, \alpha)$

$\frac{d^2x_1'(t,\beta)}{dt^2} = kx_1'(t, \beta)$

$\frac{d^2x_2'(t,\beta)}{dt^2} = kx_2'(t, \beta)$

With boundary conditions
$x_1(0, \alpha) = a_1 + \frac{\alpha l_{\widetilde{a}}}{\omega}$, $x_2(0, \alpha) = a_4 - \frac{\alpha r_{\widetilde{a}}}{\omega}$, $x_1'(0, \beta) = a_2 - \frac{\beta l_{\widetilde{a}}'}{\sigma}$, $x_2'(0, \beta) = a_3 + \frac{\beta r_{\widetilde{a}}'}{\sigma}$

and $x_1(L, \alpha) = b_1 + \frac{\alpha l_{\widetilde{b}}}{\omega}$, $x_2(L, \alpha) = b_4 - \frac{\alpha r_{\widetilde{b}}}{\omega}$, $x_1'(L, \beta) = b_2 - \frac{\beta l_{\widetilde{b}}'}{\sigma}$, $x_2'(L, \beta) = b_3 + \frac{\beta r_{\widetilde{b}}'}{\sigma}$

Where, $l_{\widetilde{a}} = a_2 - a_1$,$r_{\widetilde{a}} = a_4 - a_3$, $l_{\widetilde{a}}' = a_2 - a_1'$, $r_{\widetilde{a}}' = a_4' - a_3$
and $l_{\widetilde{b}} = b_2 - b_1$,$r_{\widetilde{b}} = b_4 - b_3$, $l_{\widetilde{b}}' = b_2 - b_1'$, $r_{\widetilde{b}}' = b_4' - b_3$
and $\omega = min\{\omega_1, \omega_2\}$, $\sigma = min\{\sigma_1, \sigma_2\}$.

Solution: The general solution of first equation is
$x_1(t, \alpha) = c_1e^{\sqrt{k}t} + c_2e^{-\sqrt{k}t}$

Using initial condition we have

$c_1 + c_2 = a_1 + \frac{\alpha l_{\widetilde{a}}}{\omega}$ and $c_1e^{\sqrt{k}L} + c_2e^{-\sqrt{k}L} = b_1 + \frac{\alpha l_{\widetilde{b}}}{\omega}$

Solving we get

$c_1 = \frac{1}{e^{\sqrt{k}L} - e^{-\sqrt{k}L}}\{(b_1 + \frac{\alpha l_{\widetilde{b}}}{\omega}) - (a_1 + \frac{\alpha l_{\widetilde{a}}}{\omega})e^{-\sqrt{k}L}\}$

and

$c_2 = -\frac{1}{e^{\sqrt{k}L}-e^{-\sqrt{k}L}}\{(b_1 + \frac{\alpha l_{\tilde{b}}}{\omega}) - (a_1 + \frac{\alpha l_{\tilde{a}}}{\omega})e^{\sqrt{k}L}\}$

Therefore the solution is

$x_1(t,\alpha) = \frac{1}{e^{\sqrt{k}L}-e^{-\sqrt{k}L}}[\{(b_1 + \frac{\alpha l_{\tilde{b}}}{\omega}) - (a_1 + \frac{\alpha l_{\tilde{a}}}{\omega})e^{-\sqrt{k}L}\}e^{\sqrt{k}t} - \{(b_1 + \frac{\alpha l_{\tilde{b}}}{\omega}) - (a_1 + \frac{\alpha l_{\tilde{a}}}{\omega})e^{\sqrt{k}L}\}e^{-\sqrt{k}t}]$

Similarly

$x_2(t,\alpha) = \frac{1}{e^{\sqrt{k}L}-e^{-\sqrt{k}L}}[\{(b_4 - \frac{\alpha r_{\tilde{b}}}{\omega}) - (a_4 - \frac{\alpha r_{\tilde{a}}}{\omega})e^{-\sqrt{k}L}\}e^{\sqrt{k}t} - \{(b_4 - \frac{\alpha r_{\tilde{b}}}{\omega}) - (a_4 - \frac{\alpha r_{\tilde{a}}}{\omega})e^{\sqrt{k}L}\}e^{-\sqrt{k}t}]$

$x_1'(t,\beta) = \frac{1}{e^{\sqrt{k}L}-e^{-\sqrt{k}L}}[\{(b_2 - \frac{\beta l'_{\tilde{b}}}{\sigma}) - (a_2 - \frac{\beta l'_{\tilde{a}}}{\sigma})e^{-\sqrt{k}L}\}e^{\sqrt{k}t} - \{(b_2 - \frac{\beta l'_{\tilde{b}}}{\sigma}) - (a_2 - \frac{\beta l'_{\tilde{a}}}{\sigma})e^{\sqrt{k}L}\}e^{-\sqrt{k}t}]$

$x_2'(t,\beta) = \frac{1}{e^{\sqrt{k}L}-e^{-\sqrt{k}L}}[\{(b_3 + \frac{\beta l'_{\tilde{b}}}{\sigma}) - (a_3 + \frac{\beta l'_{\tilde{a}}}{\sigma})e^{-\sqrt{k}L}\}e^{\sqrt{k}t} - \{(b_3 + \frac{\beta l'_{\tilde{b}}}{\sigma}) - (a_3 + \frac{\beta l'_{\tilde{a}}}{\sigma})e^{\sqrt{k}L}\}e^{-\sqrt{k}t}]$

Case 3.1.2: When $x(t)$ is (ii)-gH differentiable and $\frac{dx(t)}{dt}$ is (i)-gH then we have

$\frac{d^2x_2(t,\alpha)}{dt^2} = kx_1(t,\alpha)$

$\frac{d^2x_1(t,\alpha)}{dt^2} = kx_2(t,\alpha)$

$\frac{d^2x_2'(t,\beta)}{dt^2} = kx_1'(t,\beta)$

$\frac{d^2x_1'(t,\beta)}{dt^2} = kx_2'(t,\beta)$

With same boundary conditions.

Solution: The general solution is given by

$x_1(t,\alpha) = c_1e^{\sqrt{k}t} + c_2e^{-\sqrt{k}t} + c_3\cos\sqrt{k}t + c_4\sin\sqrt{k}t$

$x_2(t,\alpha) = c_1e^{\sqrt{k}t} + c_2e^{-\sqrt{k}t} - c_3\cos\sqrt{k}t - c_4\sin\sqrt{k}t$

$x_1'(t,\beta) = d_1e^{\sqrt{k}t} + d_2e^{-\sqrt{k}t} + d_3\cos\sqrt{k}t + d_4\sin\sqrt{k}t$

$x_2'(t,\beta) = d_1e^{\sqrt{k}t} + d_2e^{-\sqrt{k}t} - d_3\cos\sqrt{k}t - d_4\sin\sqrt{k}t$

Where, $c_1 = \frac{1}{e^{\sqrt{k}L}-e^{-\sqrt{k}L}}[\{\frac{b_1+b_4}{2} + \frac{\alpha(l_{\tilde{b}}-r_{\tilde{b}})}{2\omega}\} - \{\frac{a_1+a_4}{2} + \frac{\alpha(l_{\tilde{a}}-r_{\tilde{a}})}{2\omega}\}e^{-\sqrt{k}L}]$

$c_2 = -\frac{1}{e^{\sqrt{k}L}-e^{-\sqrt{k}L}}[\{\frac{b_1+b_4}{2} + \frac{\alpha(l_{\tilde{b}}-r_{\tilde{b}})}{2\omega}\} - \{\frac{a_1+a_4}{2} + \frac{\alpha(l_{\tilde{a}}-r_{\tilde{a}})}{2\omega}\}e^{\sqrt{k}L}]$

$c_3 = \frac{a_1-a_4}{2} + \frac{\alpha(l_{\tilde{b}}+r_{\tilde{b}})}{2\omega}$

$c_4 = \frac{1}{\sin\sqrt{k}L}[\{\frac{b_1+b_4}{2} + \frac{\alpha(l_{\tilde{b}}-r_{\tilde{b}})}{2\omega}\} - \{\frac{a_1+a_4}{2} + \frac{\alpha(l_{\tilde{a}}-r_{\tilde{a}})}{2\omega}\}\cos\sqrt{k}L]$

$d_1 = \frac{1}{e^{\sqrt{k}L}-e^{-\sqrt{k}L}}[\{\frac{b_2+b_3}{2} - \frac{\beta(l'_{\tilde{b}}-r'_{\tilde{b}})}{2\sigma}\} - \{\frac{a_2+a_3}{2} - \frac{\beta(l'_{\tilde{a}}-r'_{\tilde{a}})}{2\sigma}\}e^{-\sqrt{k}L}]$

$d_2 = -\frac{1}{e^{\sqrt{k}L}-e^{-\sqrt{k}L}}[\{\frac{b_2+b_3}{2} - \frac{\beta(l'_{\tilde{b}}-r'_{\tilde{b}})}{2\sigma}\} - \{\frac{a_2+a_3}{2} - \frac{\beta(l'_{\tilde{a}}-r'_{\tilde{a}})}{2\sigma}\}e^{\sqrt{k}L}]$

$d_3 = \frac{a_2-a_3}{2} - \frac{\beta(l'_{\tilde{a}}+r'_{\tilde{a}})}{2\sigma}$

$d_4 = \frac{1}{\sin\sqrt{k}L}[\{\frac{b_2-b_3}{2} - \frac{\beta(l'_{\tilde{b}}-r'_{\tilde{b}})}{2\sigma}\} - \{\frac{a_2-a_3}{2} - \frac{\beta(l'_{\tilde{a}}+r'_{\tilde{a}})}{2\sigma}\}\cos\sqrt{k}L]$

Case 3.1.3: When $x(t)$ is (i)-gH differentiable and $\frac{dx(t)}{dt}$ is (ii)-gH then we have

$\frac{d^2x_2(t,\alpha)}{dt^2} = kx_1(t,\alpha)$

$\frac{d^2x_1(t,\alpha)}{dt^2} = kx_2(t,\alpha)$

$\frac{d^2x'_2(t,\beta)}{dt^2} = kx'_1(t,\beta)$

$\frac{d^2x'_1(t,\beta)}{dt^2} = kx'_2(t,\beta)$

With same boundary conditions.

Solution: The result is same as Case 3.1.2.

Case 3.1.4: When $x(t)$ is (ii)-gH differentiable and $\frac{dx(t)}{dt}$ is (ii)-gH then we have

$\frac{d^2x_1(t,\alpha)}{dt^2} = kx_1(t,\alpha)$

$\frac{d^2x_2(t,\alpha)}{dt^2} = kx_2(t,\alpha)$

$\frac{d^2x'_1(t,\beta)}{dt^2} = kx'_1(t,\beta)$

$\frac{d^2x'_2(t,\beta)}{dt^2} = kx'_2(t,\beta)$

With same boundary conditions.

Solution: The result is same as Case 3.1.1.

3.2 *Second order intutionistic fuzzy boundary value problem with coefficients is negative constant i.e., $k < 0$, Consider $k = -m$, $m > 0$:*

In this problem four cases arise

Case 3.2.1 When $x(t)$ is (i)-gH differentiable and $\frac{dx(t)}{dt}$ is (i)-gH then we have

$\frac{d^2x_1(t,\alpha)}{dt^2} = -mx_2(t,\alpha)$

$\frac{d^2x_2(t,\alpha)}{dt^2} = -mx_1(t,\alpha)$

$\frac{d^2x_1'(t,\beta)}{dt^2} = -mx_2'(t,\beta)$

$\frac{d^2x_2'(t,\beta)}{dt^2} = -mx_1'(t,\beta)$

With boundary conditions

$x_1(0,\alpha) = a_1 + \frac{\alpha l_{\tilde{a}}}{\omega}$, $x_2(0,\alpha) = a_4 - \frac{\alpha r_{\tilde{a}}}{\omega}$, $x_1'(0,\beta) = a_2 - \frac{\beta l_{\tilde{a}}'}{\sigma}$, $x_2'(0,\beta) = a_3 + \frac{\beta r_{\tilde{a}}'}{\sigma}$

and $x_1(L,\alpha) = b_1 + \frac{\alpha l_{\tilde{b}}}{\omega}$, $x_2(L,\alpha) = b_4 - \frac{\alpha r_{\tilde{b}}}{\omega}$, $x_1'(L,\beta) = b_2 - \frac{\beta l_{\tilde{b}}'}{\sigma}$, $x_2'(L,\beta) = b_3 + \frac{\beta r_{\tilde{b}}'}{\sigma}$

Where, $l_{\tilde{a}} = a_2 - a_1$,$r_{\tilde{a}} = a_4 - a_3$, $l_{\tilde{a}}' = a_2 - a_1'$, $r_{\tilde{a}}' = a_4' - a_3$
and $l_{\tilde{b}} = b_2 - b_1$,$r_{\tilde{b}} = b_4 - b_3$, $l_{\tilde{b}}' = b_2 - b_1'$, $r_{\tilde{b}}' = b_4' - b_3$
and $\omega = min\{\omega_1, \omega_2\}$, $\sigma = min\{\sigma_1, \sigma_2\}$.

Solution: The general solution of this equation may be written as
$x_1(t,\alpha) = c_1 e^{\sqrt{m}t} + c_2 e^{-\sqrt{m}t} + c_3 \cos\sqrt{m}t + c_4 \sin\sqrt{m}t$

$x_2(t,\alpha) = -c_1 e^{\sqrt{m}t} - c_2 e^{-\sqrt{m}t} + c_3 \cos\sqrt{m}t + c_4 \sin\sqrt{m}t$

$x_1'(t,\beta) = d_1 e^{\sqrt{m}t} + d_2 e^{-\sqrt{m}t} + d_3 \cos\sqrt{m}t + d_4 \sin\sqrt{m}t$

$x_2'(t,\beta) = -d_1 e^{\sqrt{m}t} - d_2 e^{-\sqrt{m}t} + d_3 \cos\sqrt{m}t + d_4 \sin\sqrt{m}t$

Where, $c_1 = \frac{1}{e^{\sqrt{m}L} - e^{-\sqrt{m}L}}[\{\frac{b_1-b_2}{2} + \frac{\alpha(l_{\tilde{b}}+r_{\tilde{b}})}{2\omega}\} - \{\frac{a_1-a_4}{2} + \frac{\alpha(l_{\tilde{a}}+r_{\tilde{a}})}{2\omega}\}e^{-\sqrt{m}L}]$

$c_1 = -\frac{1}{e^{\sqrt{m}L} - e^{-\sqrt{m}L}}[\{\frac{b_1-b_4}{2} + \frac{\alpha(l_{\tilde{b}}+r_{\tilde{b}})}{2\omega}\} - \{\frac{a_1-a_4}{2} + \frac{\alpha(l_{\tilde{a}}+r_{\tilde{a}})}{2\omega}\}e^{\sqrt{m}L}]$

$c_3 = \frac{a_1+a_4}{2} + \frac{\alpha(l_{\tilde{b}}-r_{\tilde{b}})}{2\omega}$

$c_4 = \frac{1}{\sin\sqrt{m}L}[\{\frac{b_1+b_4}{2} + \frac{\alpha(l_{\tilde{b}}-r_{\tilde{b}})}{2\omega}\} - \{\frac{a_1+a_4}{2} + \frac{\alpha(l_{\tilde{a}}-r_{\tilde{a}})}{2\omega}\}\cos\sqrt{m}L]$

$d_1 = \frac{1}{e^{\sqrt{m}L} - e^{-\sqrt{m}L}}[\{\frac{b_2-b_3}{2} - \frac{\beta(l_{\tilde{b}}'+r_{\tilde{b}}')}{2\sigma}\} - \{\frac{a_2-a_3}{2} - \frac{\beta(l_{\tilde{a}}'+r_{\tilde{a}}')}{2\sigma}\}e^{-\sqrt{m}L}]$

$d_2 = -\frac{1}{e^{\sqrt{m}L} - e^{-\sqrt{m}L}}[\{\frac{b_2-b_3}{2} - \frac{\beta(l_{\tilde{b}}'+r_{\tilde{b}}')}{2\sigma}\} - \{\frac{a_2-a_3}{2} - \frac{\beta(l_{\tilde{a}}'+r_{\tilde{a}}')}{2\sigma}\}e^{\sqrt{m}L}]$

$d_3 = \frac{a_2+a_3}{2} - \frac{\beta(l_{\tilde{a}}'-r_{\tilde{a}}')}{2\sigma}$

$d_4 = \frac{1}{\sin\sqrt{m}L}[\{\frac{b_2+b_3}{2} - \frac{\beta(l_{\tilde{b}}'-r_{\tilde{b}}')}{2\sigma}\} - \{\frac{a_2+a_3}{2} - \frac{\beta(l_{\tilde{a}}'-r_{\tilde{a}}')}{2\sigma}\}\cos\sqrt{m}L]$

Case 3.2.2: When $x(t)$ is (ii)-gH differentiable and $\frac{dx(t)}{dt}$ is (i)-gH then we have

$\frac{d^2x_2(t,\alpha)}{dt^2} = -mx_2(t,\alpha)$

$\frac{d^2x_1(t,\alpha)}{dt^2} = -mx_1(t,\alpha)$

$\frac{d^2x_2^{'}(t,\beta)}{dt^2} = -mx_2^{'}(t,\beta)$

$\frac{d^2x_1^{'}(t,\beta)}{dt^2} = -mx_1^{'}(t,\beta)$

With same boundary conditions.

Solution: The solution is written as

$$x_1(t,\alpha) = (a_1 + \frac{\alpha l_{\tilde{a}}}{\omega})\cos\sqrt{m}t + \frac{1}{\sin\sqrt{m}L}\{(b_1 + \frac{\alpha l_{\tilde{b}}}{\omega}) - (a_1 + \frac{\alpha l_{\tilde{a}}}{\omega})\cos\sqrt{m}L\}\sin\sqrt{m}t$$

$$x_2(t,\alpha) = (a_4 - \frac{\alpha r_{\tilde{a}}}{\omega})\cos\sqrt{m}t + \frac{1}{\sin\sqrt{m}L}\{(b_4 - \frac{\alpha r_{\tilde{b}}}{\omega}) - (a_4 - \frac{\alpha r_{\tilde{a}}}{\omega})\cos\sqrt{m}L\}\sin\sqrt{m}t$$

$$x_1^{'}(t,\beta) = (a_3 + \frac{\beta r_{\tilde{a}}^{'}}{\sigma})\cos\sqrt{m}t + \frac{1}{\sin\sqrt{m}L}\{(b_2 - \frac{\beta l_{\tilde{b}}^{'}}{\sigma}) - (a_3 + \frac{\beta r_{\tilde{a}}^{'}}{\sigma})\cos\sqrt{m}L\}\sin\sqrt{m}t$$

$$x_2^{'}(t,\beta) = (a_2 - \frac{\beta l_{\tilde{a}}^{'}}{\sigma})\cos\sqrt{m}t + \frac{1}{\sin\sqrt{m}L}\{(b_3 + \frac{\beta r_{\tilde{b}}^{'}}{\sigma}) - (a_2 - \frac{\beta l_{\tilde{a}}^{'}}{\sigma})\cos\sqrt{m}L\}\sin\sqrt{m}t$$

Case 3.2.3: When $x(t)$ is (i)-gH differentiable and $\frac{dx(t)}{dt}$ is (ii)-gH then we have

$\frac{d^2x_2(t,\alpha)}{dt^2} = -mx_2(t,\alpha)$

$\frac{d^2x_1(t,\alpha)}{dt^2} = -mx_1(t,\alpha)$

$\frac{d^2x_2^{'}(t,\beta)}{dt^2} = -mx_2^{'}(t,\beta)$

$\frac{d^2x_1^{'}(t,\beta)}{dt^2} = -mx_1^{'}(t,\beta)$

With same boundary conditions.

Solution: The result is same as Case 3.2.2.

Case 3.2.4: When $x(t)$ is (ii)-gH differentiable and $\frac{dx(t)}{dt}$ is (ii)-gH differentiable then we have

$\frac{d^2x_1(t,\alpha)}{dt^2} = -mx_2(t,\alpha)$

$\frac{d^2x_2(t,\alpha)}{dt^2} = -mx_1(t,\alpha)$

$\frac{d^2x_1^{'}(t,\beta)}{dt^2} = -mx_2^{'}(t,\beta)$

$\frac{d^2x_2^{'}(t,\beta)}{dt^2} = -mx_1^{'}(t,\beta)$

With boundary conditions

Solution: The result is same as Case 3.2.1.

Table 1. Solutions for t=1

α	$x_{(t,\alpha)}$	$x_2(t,\alpha)$	β	$x_1^{'}(t,\beta)$	$x_2^{'}(t,\beta)$
0	1.9271	2.7245	0.2	1.7942	2.8574
0.1	1.9461	2.7055	0.3	1.6613	2.9903
0.2	1.9650	2.6865	0.4	1.5284	3.1232
0.3	1.9840	2.6675	0.5	1.3955	3.2561
0.4	2.0030	2.6485	0.6	1.2626	3.3890
0.5	2.0220	2.6295	0.7	1.1297	3.5219
0.6	2.0410	2.6106	0.8	0.9968	3.6548
0.7	2.0600	2.5916	0.9	0.8639	3.7877
			1	0.7310	3.9206

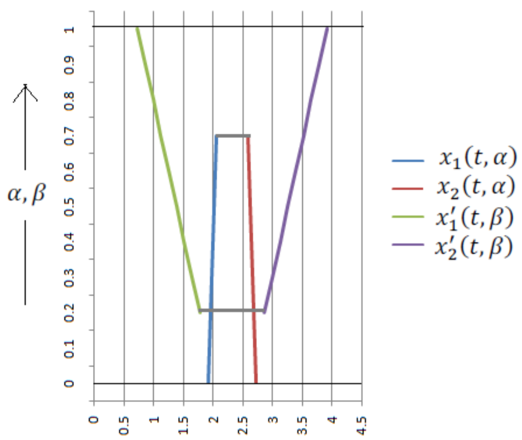

Figure 2. Graph of solutions for t=1

4. Numerical example:

Consider the differential equation $\frac{d^2x(t)}{dt^2} = 4x(t)$, with boundary conditions $x(0) = ((4.5, 5, 7, 7.5; 0.7), (4, 5, 7, 8; 0.2))$
and $x(2) = ((10, 10.5, 12.5, 13; 0.7), (9.5, 10.5, 12.5, 13.5; 0.2))$
Find the solution when $x(t)$ and $\frac{dx(t)}{dt}$ is (i)-gH differentiable.

Solution: The solution is given by

$$x_1(t,\alpha)=\frac{1}{e^4-e^{-4}}[\{(10+\tfrac{5\alpha}{7})-(4.5+\tfrac{5\alpha}{7})e^{-4}\}e^{2t}-\{(10+\tfrac{5\alpha}{7})-(4.5+\tfrac{5\alpha}{7})e^{4}\}e^{-2t}]$$

$$x_2(t,\alpha)=\frac{1}{e^4-e^{-4}}[\{(13-\tfrac{5\alpha}{7})-(7.5-\tfrac{5\alpha}{7})e^{-4}\}e^{2t}-\{(13-\tfrac{5\alpha}{7})-(7.5-\tfrac{5\alpha}{7})e^{4}\}e^{-2t}]$$

$$x_1^{'}(t,\beta)=\frac{1}{e^4-e^{-4}}[\{(10.5-5\beta)-(5-5\beta)e^{-4}\}e^{2t}-\{(10.5-5\beta)-(5-5\beta)e^{4}\}e^{-2t}]$$

$$x_2^{'}(t,\beta)=\frac{1}{e^4-e^{-4}}[\{(12.5+5\beta)-(7+5\beta)e^{-4}\}e^{2t}-\{(12.5+5\beta)-(7+5\beta)e^{4}\}e^{-2t}]$$

Remarks: From the graph and table we conclude that $x_1(t,\alpha)$ is increasing and $x_2(t,\alpha)$ is decreasing whereas $x_1^{'}(t,\beta)$ is decreasing and $x_2^{'}(t,\beta)$ is increasing function for $t = 1$. Thus in this case the solution is a strong solution.

5. Conclusion:

In this paper second order intutionistic fuzzy boundary value problem is solved. The intutionistic fuzzy number is taken as generalized trapezoidal intuitionistic fuzzy number. The coefficient is positive crisp number and negative crisp number are taken in this paper and solved the problem analytically. Finally a numerical example is illustrated. In future we can apply this procedure in n-th order intutionistic fuzzy boundary value problem and use the results in different applications in the field of sciences and engineering.

Acknowledgement

We are grateful to the reviewers for their careful reading, valuable comments and helpful suggestions which enable to improve the presentation of this paper significantly.

References

[1] L. A. Zadeh, Fuzzy sets, Information and Control, 8, (1965) 338-353.
[2] D.Dubois, H.Parade, Operation on Fuzzy Number, International Journal of Fuzzy system, 9, (1978) 613-626.
[3] K. T. Atanassov, Intuitionistic fuzzy sets, VII ITKRs Session, Sofia, Bulgarian, 1983.
[4] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, Vol. 20, (1986) 8796.
[5] L. A. Zadeh, Toward a generalized theory of uncertainty (GTU) an outline, Information Sciences 172, (2005) 140.
[6] S. L. Chang, L. A. Zadeh, On fuzzy mapping and control, IEEE Transaction on Systems Man Cybernetics 2, (1972) 3034.
[7] D. Dubois, H. Prade, Towards fuzzy differential calculus: Part 3, Differentiation, Fuzzy Sets and Systems 8, (1982) 225233.
[8] M. L. Puri, D. A. Ralescu, Differentials of fuzzy functions, Journal of Mathematical Analysis and Application 91, (1983) 552558.
[9] R. Goetschel, W. Voxman, Elementary fuzzy calculus, Fuzzy Sets and Systems 18, (1986) 3143.
[10] O. Kaleva, Fuzzy differential equations, Fuzzy Sets and Systems 24, (1987) 301317.
[11] B. Bede, A note on two-point boundary value problems associated with non-linear fuzzy differential equations, Fuzzy Sets. Syst.157, (2006) 986989.
[12] B. Bede,S. G. Gal, Generalizations of the differentiability of fuzzy-number-valued functions with applications to fuzzy differential equations, Fuzzy Sets Syst.151, (2005) 581599.
[13] Y. Chalco-Cano, H. Romn-Flores, On the new solution of fuzzy differential equations, Chaos Solitons Fractals 38, (2008) 112119.
[14] B. Bede,I. J. Rudas and A. L. Bencsik, First order linear fuzzy differential equations under generalized differentiability, Inf. Sci. 177, (2007) 16481662.
[15] Y. Chalco-Cano, M.A.Rojas-Medar,H.Romn-Flores, Sobre ecuaciones diferencial esdifusas, Bol. Soc. Esp. Mat. Apl. 41, (2007) 9199.
[16] Y. Chalco-Cano, H. Romn-Flores and M. A. Rojas-Medar, Fuzzy differential equations with generalized derivative, in:Proceedings of the 27th North American Fuzzy Information Processing Society International Conference, IEEE, 2008.
[17] L. Stefanini, B. Bede, Generalized Hukuhara differentiability of interval-valued functions and interval differential equations, Nonlinear Anal. 71, (2009) 13111328.
[18] A. Khastan,J. J. Nieto, A boundary value problem for second-order fuzzy differential equations, Nonlinear Anal. 72, (2010) 35833593.
[19] P. Diamond, P. Kloeden, Metric Spaces of Fuzzy Sets, World Scientific, Singapore, 1994.
[20] B. Bede, S. G. Gal, Generalizations of the differentiability of fuzzy-number-valued functions with applications to fuzzy differential equations, Fuzzy Set Systems, 151 (2005) 581-599.
[21] L. Stefanini, A generalization of Hukuhara difference for interval and fuzzy arithmetic, in: D. Dubois, M.A. Lubiano, H. Prade, M. A. Gil, P. Grzegorzewski, O. Hryniewicz (Eds.), Soft Methods for Handling Variability and Imprecision, in: Series on Advances in Soft Computing, 48 (2008).
[22] L. Stefanini, B. Bede, Generalized Hukuhara differentiability of interval-valued functions and interval differential equations, Nonlinear Analysis, 71 (2009) 1311-1328.
[23] A. Armand, Z. Gouyandeh, Solving two-point fuzzy boundary value problem using the variational iteration method, Communications on Advanced Computational Science with Applications, Vol. 2013, (2013) 1-10.
[24] N. Gasilov, S. E. Amrahov, A. G. Fatullayev, Solution of linear differential equations with fuzzy boundary values, Fuzzy Sets and Systems 257, (2014) 169-183.

[25] N. Gasilov, S. E. Amrahov, A. G. Fatullayev, A. Khastan, A new approach to fuzzy initial value problem ,18 (2), (2014) 217-225.
[26] B. Bede, L. Stefanini, Generalized differentiability of fuzzy-valued functions, Fuzzy Sets and Systems, 230, (2013) 119-141.
[27] R. Rodrguez-Lpez, On the existence of solutions to periodic boundary value problems for fuzzy linear differential equations, Fuzzy Sets and Systems, 219, (2013) 1-26.
[28] S. Melliani, L. S. Chadli, Introduction to intuitionistic fuzzy partial differential Equations, Fifth Int. Conf. on IFSs, Sofia, 22-23 Sept. 2001.
[29] S. Abbasbandy, T. Allahviranloo, Numerical Solution of Fuzzy Differential Equations by Runge-Kutta and the Intuitionistic Treatment, Journal of Notes on Intuitionistic Fuzzy Sets, Vol. 8, No. 3, (2002) 43-53.
[30] S. Lata, A.Kumar, A new method to solve time-dependent intuitionistic fuzzy differential equation and its application to analyze the intutionistic fuzzy reliability of industrial system, Concurrent Engineering: Research and Applications, (2012) 1-8.
[31] S. P. Mondal and T. K. Roy, First order homogeneous ordinary differential equation with initial value as triangular intuitionistic fuzzy number, Journal of Uncertainty in Mathematics Science (2014) 1-17.
[32] S. P. Mondal. and T. K. Roy, System of Differential Equation with Initial Value as Triangular Intuitionistic Fuzzy Number and its Application, Int. J. Appl. Comput. Math, (2010).
[33] L. C. Barros, L. T. Gomes, P. A Tonelli, Fuzzy differential equations: An approach via fuzzification of the derivative operator, Fuzzy Sets and Systems, 230, (2013) 39-52.
[34] M.R.Seikh, P.K.Nayak and M.Pal, Generalized Triangular Fuzzy Numbers In Intuitionistic Fuzzy Environment, International Journal of Engineering Research and Development, Volume 5, Issue 1 (2012) 08-13.
[35] H.J.Zimmerman, Fuzzy set theory and its applications, Kluwer Academi Publishers, Dordrecht (1991).