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Coupled fixed point theorems involving contractive condition of integral type in generalized metric spaces

R. Shah[a*], A. Zada[a]

[a]*Department of Mathematics, University of Peshawar, Peshawar, Pakistan.*

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Abstract. In this manuscript, we prove some coupled fixed point theorems for two pairs of self mappings satisfying contractive conditions of integral type in generalized metric spaces. We furnish suitable illustrative examples.

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1. Introduction

The study of common and coincidence fixed points of mappings is one of the central research area see, [1, 2, 4, 6, 9–13, 15, 17, 18]. In 2006, Mustafa and Sims [16], introduced the concept of G-metric space and presented some fixed point theorems in G-metric space. The concept of a coupled coincidence point of mapping was introduced by Lakshmikantham et al. [7, 14], they also studied some fixed point theorems in partially ordered metric spaces. In 2010, Shatanawi [23] gave the proof of coupled coincidence fixed point theorems in generalized metric spaces. Moreover, in 2002, Branciari [8] gave the idea of integral type contractive mappings in complete metric spaces and studied the existence of fixed points for mappings which is defined on complete metric space satisfies integral type contraction. Recently, Shah et al. [21, 22, 24], presented the concept of integral type contraction in generalized metric spaces and proved some coupled coincidence fixed point results for two pairs in such spaces, by using the notion of integral type contractive

*Corresponding author.
E-mail address: safeer_rahim@yahoo.com (R. Shah).

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mappings given by Branciari [8]. In the main section, we present some coupled fixed point theorems of integral type contractive mappings in setting of G-metric spaces. Finally, we present some examples. We recommend some other references to the reader see [5, 19, 20].

2. Preliminaries

We needs the following definitions and results in this paper.

Definition 2.1 [16] Let Y be a non-empty set and $G : Y \times Y \times Y \to R^+$ is a function that satisfies the following conditions:

(1) $G(a, a, b) > 0$ for all $a, b \in Y$ with $a \neq b$,
(2) $G(a, b, c) = 0$ if and only if $a = b = c$,
(3) $G(a, a, b) \leqslant G(a, b, c)$, for all $a, b, c \in Y$ with $c \neq b$
(4) $G(a, b, c) = G(a, c, b) = G(b, c, a) = \ldots$, symmetry in all variables,
(5) $G(a, b, c) \leqslant G(a, s, s) + G(s, b, c)$ for all $a, b, c, s \in Y$.

Then the function G is called a generalized metric and the pair (Y, G) is called a G-metric space.

***Example* 2.2** [16] Let $Y = \{x, y\}$. Define G on $Y \times Y \times Y$ by

$$G(x, x, x) = G(y, y, y) = 0, G(x, x, y) = 1, G(x, y, y) = 2$$

and extend G to $Y \times Y \times Y$ by using the symmetry in the variables. Then it is clear that (Y, G) is a G-metric space.

Definition 2.3 [16] Let (Y, G) be a G-metric space and (a_n) a sequence of points of Y. A point $a \in Y$ is said to be the limit of the sequence (a_n), if $\lim_{n,m\to+\infty} G(a, a_n, a_m) = 0$, and we say that the sequence (a_n) is G-convergent to a.

Proposition 2.4 [16] Let (Y, G) be a G-metric space. Then the following are equivalent:

(1) (a_n) is G-convergent to a.
(2) $G(a_n, a_n, a) \to 0$ as $n \to +\infty$.
(3) $G(a_n, a, a) \to 0$ as $n \to +\infty$.
(4) $G(a_n, a_m, a) \to 0$ as $n, m \to +\infty$.

Definition 2.5 [15] Let (Y, G) be a G-metric space. A sequence (a_n) a sequence is called G-Cauchy if for every $\epsilon > 0$, there is $k \in \mathbf{N}$ such that $G(a_n, a_m, a_l) < \epsilon$, for all $n, m, l \geqslant k$; that is $G(a_n, a_m, a_l) \to 0$ as $n, m, l \to +\infty$.

Proposition 2.6 [16] Let (Y, G) be a G-metric space. Then the following are equivalent:

(1) The sequence (a_n) is G-Cauchy.
(2) For every $\epsilon > 0$, there is $k \in \mathbf{N}$ such that $G(a_n, a_m, a_m) < \epsilon$, for all $n, m \geqslant k$.

Definition 2.7 [16] A G-metric space (Y, G) is called G-complete if every G-Cauchy sequence in (Y, G) is G-convergent in (Y, G).

Definition 2.8 [7] An element $(a, b) \in Y \times Y$ is called a coupled coincidence point of mapping $F : Y \times Y \to Y$ and $g : Y \to Y$ if $F(a, b) = ga$ and $F(b, a) = gb$.

Definition 2.9 [14] let Y be a non-empty set. Then we say that the mappings $F : Y \times Y \to Y$ and $g : Y \to Y$ are commutative if $gF(a, b) = F(ga, gb)$.

Definition 2.10 [14] An element $(a, b) \in Y \times Y$ is called a coupled fixed point of mapping $F : Y \times Y \to Y$ if $F(a, b) = a$ and $F(b, a) = b$.

In 2002, Branciari in [8] gave the notion of integral type contractive mappings and introduced a general contractive condition of integral type as follows.

Theorem 2.11 [8] Let (Y, d) be a complete metric space, $\alpha \in (0, 1)$, and $f : Y \to Y$ is a mapping such that for all $x, y \in Y$,

$$\int_0^{d(f(x),f(y))} \phi(t)dt \leqslant \alpha \int_0^{d(x,y)} \phi(t)dt,$$

where $\phi : [0, +\infty) \to [0, +\infty)$ is nonnegative and Lebesgue-integrable mapping which is summable (i.e., with finite integral) on each compact subset of $[0, +\infty)$ such that for each $\epsilon > 0$, $\int_0^\epsilon \phi(t)dt > 0$, then f has a unique fixed point $a \in Y$, such that for each $x \in Y$, $\lim_{n\to\infty} f^n(x) = a$.

By using the above idea of integral type contraction given by Branciari in [8], we presented our results in generalized metric spaces.

3. Main Results

In this section, we prove some coupled fixed point theorems in generalized metric space by using integral type contractive mapping. We start with the following definition, which will be essential for the proof of the main theorems.

Definition 3.1 [3] . $\varphi : [0, +\infty) \to [0, +\infty)$ is subadditive on each $[a, b] \subset [0, +\infty)$ if,

$$\int_0^{a+b} \varphi(t)dt \leqslant \int_0^a \varphi(t)dt + \int_0^b \varphi(t)dt.$$

We start our work by following important lemma.

Lemma 3.2 Let (Y, G) be a G-metric space. Suppose $F : Y \times Y \to Y$ and $g : Y \to Y$ be two mappings such that

$$\int_0^{G(F(a,b),F(p,q),F(c,r))} \varphi(t)dt \leqslant k \int_0^{(G(ga,gp,gc)+G(gb,gq,gr))} \varphi(t)dt, \tag{1}$$

for all $a, b, c, p, q, r \in Y$ and $\varphi : [0, \infty) \to [0, \infty)$ is a Lebesgue integrable mapping which is summable, non-negative and such that for each $\epsilon > 0$, $\int_0^\epsilon \varphi(t)dt > 0$. Assume that (a, b) is coupled coincidence point of mappings F and g. If $k \in [0, \frac{1}{2})$, then $F(a, b) = ga = gb = F(b, a)$.

Proof. Since (a, b) is a coupled coincidence point of the mappings F and g, we have $ga = F(a, b)$ and $gb = F(b, a)$. Suppose $ga \neq gb$. Then by (1), we get

$$\begin{aligned}\int_0^{G(ga,gb,gb)} \varphi(t)dt &= \int_0^{G(F(a,b),F(b,a),F(b,a))} \varphi(t)dt \\ &\leqslant k \int_0^{(G(ga,gb,gb)+G(gb,ga,ga))} \varphi(t)dt.\end{aligned}$$

Also we have,

$$\begin{aligned}\int_0^{G(gb,ga,ga)} \varphi(t)dt &= \int_0^{G(F(b,a),F(a,b),F(a,b))} \varphi(t)dt \\ &\leqslant k \int_0^{(G(gb,ga,ga)+G(ga,gb,gb))} \varphi(t)dt.\end{aligned}$$

Therefore

$$\int_0^{G(ga,gb,gb)} \varphi(t)dt + \int_0^{G(gb,ga,ga)} \varphi(t)dt \leqslant 2k \int_0^{(G(ga,gb,gb)+G(gb,ga,ga))} \varphi(t)dt.$$

Since $2k < 1$ and by Definition 3.1, we can write

$$\int_0^{G(ga,gb,gb)} \varphi(t)dt + \int_0^{G(gb,ga,ga)} \varphi(t)dt < \int_0^{G(ga,gb,gb)} \varphi(t)dt + \int_0^{G(gb,ga,ga)} \varphi(t)dt,$$

which is contradiction. So $ga = gb$, and hence $F(a,b) = ga = gb = F(b,a)$. The proof is completed. ■

Theorem 3.3 Let (Y, G) be a G-metric space. let $F : Y \times Y \to Y$ and $g : Y \to Y$ be two mappings such that

$$\int_0^{G(F(a,b),F(p,q),F(c,r))} \varphi(t)dt \leqslant k \int_0^{(G(ga,gp,gc)+G(gb,gq,gr))} \varphi(t)dt \tag{2}$$

for all $a, b, c, p, q, r \in Y$ and $\varphi : [0,\infty) \to [0,\infty)$ is a Lebesgue integrable mapping which is summable, non-negative and such that for each $\epsilon > 0$, $\int_0^{\epsilon} \varphi(t)dt > 0$. Assume that F and g satisfy the following conditions:

(i) $F(Y \times Y) \subseteq g(Y)$;
(ii) $g(Y)$ is complete and;
(iii) g is G-continuous and commutes with F.

If $k \in [0, \frac{1}{2})$, then there is a unique $a \in Y$ such that $ga = F(a,a) = a$.

Proof. Let $a_0, b_0 \in Y$. Since $F(Y \times Y) \subseteq g(Y)$, choose $a_1, b_1 \in Y$ such that $ga_1 = F(a_0, b_0)$ and $gb_1 = F(b_0, a_0)$. Again since $F(Y \times Y) \subseteq g(Y)$, choose $a_2, b_2 \in Y$ such that $ga_2 = F(a_1, b_1)$ and $gb_2 = F(b_1, a_1)$. Continuing this process, we can construct two sequences (a_n) and (b_n) in Y such that $ga_{n+1} = F(a_n, b_n)$ and $gb_{n+1} = F(b_n, a_n)$. For $n \in N$, we have

$$\begin{aligned}\int_0^{G(ga_n,ga_{n+1},ga_{n+1})} \varphi(t)dt &= \int_0^{(G(F(a_{n-1},b_{n-1}),F(a_n,b_n),F(a_n,b_n))} \varphi(t)dt \\ &\leqslant k \int_0^{(G(ga_{n-1},ga_n,ga_n)+G(gb_{n-1},gb_n,gb_n))} \varphi(t)dt.\end{aligned}$$

From

$$\begin{aligned}\int_0^{G(ga_{n-1},ga_n,ga_n)}\varphi(t)dt &= \int_0^{(G(F(a_{n-2},b_{n-2}),F(a_{n-1},b_{n-1}),F(a_{n-1},b_{n-1}))}\varphi(t)dt\\ &\leqslant k\int_0^{(G(ga_{n-2},ga_{n-1},ga_{n-1})+G(gb_{n-2},gb_{n-1},gb_{n-1}))}\varphi(t)dt,\end{aligned}$$

and

$$\begin{aligned}\int_0^{G(gb_{n-1},gb_n,gb_n)}\varphi(t)dt &= \int_0^{(G(F(b_{n-2},a_{n-2}),F(b_{n-1},a_{n-1}),F(b_{n-1},a_{n-1}))}\varphi(t)dt\\ &\leqslant k\int_0^{(G(gb_{n-2},gb_{n-1},gb_{n-1})+G(ga_{n-2},ga_{n-1},ga_{n-1}))}\varphi(t)dt,\end{aligned}$$

we have

$$\begin{aligned}\int_0^{G(ga_{n-1},ga_n,ga_n)}\varphi(t)dt &+ \int_0^{G(gb_{n-1},gb_n,gb_n)}\varphi(t)dt\\ &\leqslant 2k\int_0^{(G(ga_{n-2},ga_{n-1},ga_{n-1})+G(gb_{n-2},gb_{n-1},gb_{n-1}))}\varphi(t)dt\end{aligned}$$

holds for all $n \in N$. Thus, we get

$$\begin{aligned}\int_0^{G(ga_n,ga_{n+1},ga_{n+1})}\varphi(t)dt &\leqslant k\int_0^{(G(ga_{n-1},ga_n,ga_n)+G(gb_{n-1},gb_n,gb_n))}\varphi(t)dt\\ &\leqslant 2k^2\int_0^{(G(ga_{n-2},ga_{n-1},ga_{n-1})+G(gb_{n-2},gb_{n-1},gb_{n-1}))}\varphi(t)dt\\ &\vdots\\ &\leqslant \frac{1}{2}(2k)^n\int_0^{(G(ga_0,ga_1,ga_1)+G(gb_0,gb_1,gb_1))}\varphi(t)dt.\end{aligned}$$

Thus

$$\int_0^{G(ga_n,ga_{n+1},ga_{n+1})}\varphi(t)dt \leqslant \frac{1}{2}(2k)^n\int_0^{(G(ga_0,ga_1,ga_1)+G(gb_0,gb_1,gb_1))}\varphi(t)dt.$$

Let $m, n \in N$ with $m > n$ and by Definition 3.1, we can write

$$\begin{aligned}\int_0^{G(ga_n,ga_m,ga_m)}\varphi(t)dt &\leqslant \int_0^{G(ga_n,ga_{n+1},ga_{n+1})}\varphi(t)dt + \int_0^{G(ga_{n+1},ga_{n+2},ga_{n+2})}\varphi(t)dt\\ &+ \ldots + \int_0^{G(ga_{m-1},ga_m,ga_m)}\varphi(t)dt.\end{aligned}$$

Since $2k < 1$, by above inequality we get

$$\int_0^{G(ga_n,ga_m,ga_m)} \varphi(t)dt \leqslant \frac{1}{2}\Big(\sum_{i=n}^{m-1}(2k)^i\Big)\int_0^{(G(ga_0,ga_1,ga_1)+G(gb_0,gb_1,gb_1))} \varphi(t)dt$$
$$\leqslant \frac{(2k)^n}{2(1-2k)}\int_0^{(G(ga_0,ga_1,ga_1)+G(gb_0,gb_1,gb_1))} \varphi(t)dt \to 0,$$

as $n \to \infty$. Thus

$$\lim_{n,m\to\infty} G(ga_n, ga_m, ga_m) = 0.$$

Thus (ga_n) is G-Cauchy in $g(Y)$. As $g(Y)$ is G-complete, we get that (ga_n) and (gb_n) are G-convergent to some $a \in Y$ and $b \in Y$. Since g is G-continuous, we have (gga_n) is G-convergent to ga and (ggb_n) is G-convergent to gb. Also g and F commute, we have $gga_{n+1} = g(F(a_n, b_n)) = F(ga_n, gb_n)$, and $ggb_{n+1} = g(F(b_n, a_n)) = F(gb_n, ga_n)$. Thus

$$\int_0^{G(gga_{n+1},F(a,b),F(a,b))} \varphi(t)dt = \int_0^{G(F(ga_n,gb_n),F(a,b),F(a,b))} \varphi(t)dt$$
$$\leqslant k\int_0^{(G(gga_n,ga,ga)+G(ggb_n,gb,gb))} \varphi(t)dt.$$

When $n \to \infty$, also by continuity we get,

$$\int_0^{G(ga,F(a,b),F(a,b))} \varphi(t)dt \leqslant k\int_0^{(G(ga,ga,ga)+G(gb,gb,gb))} \varphi(t)dt = 0.$$

Hence we can say that $ga = F(a, b)$. By the same way, we can show that $gb = F(b, a)$. By Lemma 3.2, (a, b) is a coupled fixed point of F and g. So $ga = F(a, b) = F(b, a) = gb$. Since (ga_{n+1}) is subsequence of ga_n we have that (ga_{n+1}) is convergent to a. Thus

$$\int_0^{G(ga_{n+1},ga,ga)} \varphi(t)dt = \int_0^{G(ga_{n+1},F(a,b),F(a,b))} \varphi(t)dt$$
$$= \int_0^{G(F(a_n,b_n),F(a,b),F(a,b))} \varphi(t)dt$$
$$\leqslant k\int_0^{(G(ga_n,ga,ga)+G(gb_n,gb,gb))} \varphi(t)dt.$$

When $n \to \infty$, also by continuity we get,

$$\int_0^{G(a,ga,ga)} \varphi(t)dt \leqslant k\int_0^{(G(a,ga,ga)+G(b,gb,gb))} \varphi(t)dt.$$

In the same manner, we get

$$\int_0^{G(b,gb,gb)} \varphi(t)dt \leqslant k\int_0^{(G(a,ga,ga)+G(b,gb,gb))} \varphi(t)dt.$$

Thus

$$\int_0^{G(a,ga,ga)} \varphi(t)dt + \int_0^{G(b,gb,gb)} \varphi(t)dt \leqslant 2k \int_0^{(G(a,ga,ga)+G(b,gb,gb))} \varphi(t)dt.$$

Since $2k < 1$, the inequality happens only if $\int_0^{G(a,ga,ga)} \varphi(t)dt = 0$ and $\int_0^{G(b,gb,gb)} \varphi(t)dt = 0$. Hence $a = ga$ and $b = gb$. So we get $ga = F(a,a) = a$. For uniqueness: Let $y \in Y$ with $y \neq a$ such that $y = gy = F(y,y)$. Then

$$\begin{aligned}\int_0^{G(a,y,y)} \varphi(t)dt &= \int_0^{G(F(a,a),F(y,y),F(y,y))} \varphi(t)dt \\ &\leqslant 2k \int_0^{G(ga,gy,gy)} \varphi(t)dt \\ &= 2k \int_0^{G(a,y,y)} \varphi(t)dt.\end{aligned}$$

Since $2k < 1$, we get

$$\int_0^{G(a,y,y)} \varphi(t)dt < \int_0^{G(a,y,y)} \varphi(t)dt.$$

Which is contradiction. Thus F and g have a unique common fixed point. The proof is completed. ■

Corollary 3.4 Let (Y,G) be a G-metric space. Let $F : Y \times Y \to Y$ and $g : Y \to Y$ be two mappings such that

$$\int_0^{G(F(a,b),F(p,q),F(p,q))} \varphi(t)dt \leqslant k \int_0^{(G(ga,gp,gp)+G(gb,gq,gq))} \varphi(t)dt, \tag{3}$$

for all $a,b,p,q \in Y$ and $\varphi : [0,\infty) \to [0,\infty)$ is a Lebesgue integrable mapping which is summable, non-negative and such that for each $\epsilon > 0$, $\int_0^{\epsilon} \varphi(t)dt > 0$. Assume that F and g satisfy the following conditions:

(i) $F(Y \times Y) \subseteq g(Y)$
(ii) $g(Y)$ is complete and
(iii) g is G-continuous and commute with F.

If $k \in [0, \frac{1}{2})$, then there is a unique $a \in Y$ such that $ga = F(a,a) = a$.

Proof. In Theorem 3.3, set $c = p$ and $q = r$. ■

Corollary 3.5 Let (Y,G) be a G-metric space. Let $F : Y \times Y \to Y$ and $g : Y \to Y$ be two mappings such that

$$\int_0^{G(F(a,b),F(p,q),F(p,q))} \varphi(t)dt \leqslant k \int_0^{(G(a,p,p)+G(b,q,q))} \varphi(t)dt, \tag{4}$$

for all $a,b,p,q \in Y$ and $\varphi : [0,\infty) \to [0,\infty)$ is a Lebesgue integrable mapping which is summable, non-negative and such that for each $\epsilon > 0$, $\int_0^{\epsilon} \varphi(t)dt > 0$. If $k \in [0, \frac{1}{2})$, then there is a unique $a \in Y$ such that $ga = F(a,a) = a$.

Proof. Let us define $g : Y \to Y$ by $ga = a$. Then F and g follows all the assumptions of lemma 3.2. Hence the result follows. ■

***Example* 3.6** Let $Y = [0, 1]$. Define $G : Y \times Y \times Y \to R^+$ by

$$G(a, b, c) = |a - b| + |a - c| + |b - c|$$

for all $a, b, c \in Y$. Then (Y, G) is a complete G-metric space. Define a mapping $F : Y \times Y \to Y$ by $F(a, b) = \frac{1}{6}ab$ for $a, b \in Y$. Also, define
$g : Y \to Y$ by $g(a) = \frac{1}{2}$ for $a \in Y$. Since
$|ab - pq| \leqslant |a - p| + |b - q|$

Then the condition of main result (3.3) holds, in fact

$$\begin{aligned} \int_0^{G(F(a,b),F(p,q),F(c,r))} \varphi(t)dt &= \int_0^{G(\frac{1}{6}ab,\, \frac{1}{6}pq,\, \frac{1}{6}cr)} \varphi(t)dt \\ &\leqslant \frac{1}{3}\int_0^{(G(ga,gp,gc)+G(gb,gq,qr))} \varphi(t)dt. \end{aligned}$$

Thus F and g have a unique common fixed point. Here $F(0, 0) = g(0) = 0$.

***Example* 3.7** Let $Y = [-1, 1]$. Define $G : Y \times Y \times Y \to R^+$ by

$$G(a, b, c) = |a - b| + |a - c| + |b - c|$$

for all $a, b, c \in Y$. Then (Y, G) is a complete G-metric space. Define a mapping $F : Y \times Y \to Y$ by $F(a, b) = \frac{1}{8}a^2 + \frac{1}{8}b^2 - 1$
for all $a, b \in Y$. So, condition of Corollary (3.5) holds, we have

$$\begin{aligned} \int_0^{G(F(a,b),\, F(p,q),\, F(p,q))} \varphi(t)dt &= \int_0^{G(\frac{1}{8}a^2+\frac{1}{8}b^2-1,\, \frac{1}{8}p^2+\frac{1}{8}q^2-1,\, \frac{1}{8}p^2+\frac{1}{8}q^2-1)} \varphi(t)dt \\ &\leqslant \frac{1}{4}\int_0^{(G(a,p,p)+G(b,q,q))} \varphi(t)dt. \end{aligned}$$

Hence F has a unique fixed point. Here $a = 2 - 2\sqrt{2}$ is the unique fixed point of F, i.e. $F(a, a) = a$.

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