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Generalized inverse of fuzzy neutrosophic soft matrix

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Abstract. Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. Aim of this article is to find the maximum and minimum solution of the fuzzy neutrosophic soft relational equations $xA = b$ and $Ax = b$, where x and b are fuzzy neutrosophic soft vectors and A is a fuzzy neutrosophic soft matrix. Whenever A is singular we can not find A^{-1}. In that case we can use generalized inverse (g-inverse) to get the solution of the above said relational equations. Further, using this concept maximum and minimum g-inverse of fuzzy neutrosophic soft matrix are obtained.

Keywords: Fuzzy neutrosophic soft set (FNSS), Fuzzy neutrosophic soft matrix (FNSM), g-inverse.

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1. Introduction

Most of our real life problems in medical science, engineering, management, environment and social sciences often involve data which are not necessarily crisp, precise and deterministic in character due to various uncertainties associated with these problems. Such uncertainties are usually being handled with the help of the topics like probability, fuzzy sets, interval mathematics and rough sets etc. Intuitionistic fuzzy sets introduced by Atanassov [3] is appropriate for such a situation. The intuitionistic fuzzy sets can only handle the incomplete information considering both the truth membership and falsity

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membership. It does not handle the indeterminate and inconsistent information which exists in belief system. Florentin Smarandache [5] introduced the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. The neutrosophic components T,I, F which represents the membership, indeterminancy, and non-membership values respectively, where $]^{-}0, 1^{+}[$ is the non-standard unit interval, and thus one defines the neutrosophic set.

For example the Schrodinger's cat theory says that the quantum state of a photon can basically be in more than one place at the same time, which translated to the neutrosophic set which means an element (quantum state) belongs and does not belong to a set (one place) at the same time; or an element (quantum state) belongs to two different sets(two different places) in the same time. Diletheism is the view that some statements can be both true and false simultaneously. More precisely, it is belief that there can be true statement whose negation is also true. Such statement are called true contradiction, diletheia or nondualism. "All statements are true" is a false statement. The above example of true contradictions that dialetheists accept. Neutrosophic set, like dialetheism, can describe paradoxist elements, Neutrosophic set (paradoxist element)=(1,1,1), while intuitionistic fuzzy logic can not describe a paradox because the sum of components should be 1 in intuitionistic fuzzy set.

In neutrosophic set there is no restriction on T,I,F other than they are subsets of $]^{-}0, 1^{+}[$, thus

$$^{-}0 \leqslant infT + infI + infF \leqslant supT + supI + supF \leqslant 3^{+}$$

Neutrosophic sets and logic are the foundations for many theories which are more general than their classical counterparts in fuzzy, intuitionistic fuzzy, paraconsistent set, dialetheist set, paradoxist set and tautological set.

In 1999, Molodtsov [11] initiated the novel concept of soft set theory which is a completely new approach for modeling vagueness and uncertainty. In [9] Maji et al., initiated the concept of fuzzy soft sets with some properties regarding fuzzy soft union, intersection, complement of fuzzy soft set. Moreover in [10, 13] Maji et al., extended soft sets to intuitionistic fuzzy soft sets and neutrosophic soft sets.

One of the important theory of Mathematics which has a vast application in Science and Engineering is the theory of matrices. Let A be a square matrix of full rank. Then, there exists a matrix X such that $AX = XA = I$. This X is called the inverse of A and is denoted by A^{-1}. Suppose A is not a matrix of full rank or it is a rectangular matrix, in such a case inverse does not exists. In recent years needs have been felt in numerous areas of applied Mathematics for some kind of partial inverse of a matrix which is singular or even rectangular, such inverse are called generalized inverse. Solving fuzzy matrix equation of the type $xA = b$ where

$$x = (x_{11}, x_{12}, x_{1m}),$$
$$b = (b_{11}, b_{12}, b_{1n})$$

and A is a fuzzy matrix of order $m \times n$ is of great interest in various fields. We say $xA = b$ is computable, if there exists a solution for $xA = b$ and in this case we write

$$\max_{j} min(x_{1j}, a_{jk}) = b_{1k} \quad \forall\, j \in I_m,\ k \in I_n,$$

where I_n is an index set, $i = 1, 2, ..., n$ and $\Omega_1(A, b)$ represents the set of all solutions of

$xA = b$.

The authors extend this concept into fuzzy neutrosophic soft matrix. The fuzzy neutrosophic soft matrix equation is of the form $xA = b \cdots (1)$ where,

$$x = \langle x_{11}^T, x_{11}^I, x_{11}^F\rangle \cdots \langle x_{1m}^T, x_{1m}^I, x_{1m}^F\rangle$$
$$b = \langle b_{11}^T, b_{11}^I, b_{11}^F\rangle \cdots \langle b_{1n}^T, b_{1n}^I, b_{1n}^F\rangle,$$

and A is a fuzzy neutrosophic soft matrix of order $m \times n$. The equation $xA = b$ is computable if there exist a solution for $xA = b$ and in this case we write

$$\max_j min\langle x_{1j}^T, x_{1j}^I, x_{1j}^F\rangle . \langle a_{jk}^T, a_{jk}^I, a_{jk}^F\rangle = \langle b_{1k}^T, b_{1k}^I, b_{1k}^F\rangle \quad \forall\, j \in I_m,\ k \in I_n.$$

Denote $\Omega_1(A, b) = \{x | xA = b\}$ represents the set of all solutions of $xA = b$. Several authors [4, 8, 14] have studied about the maximum solution $\hat{x}$ and the minimum solution of $xA = b$ for fuzzy matrix as well as IFMs (Intuitionistic fuzzy matrix).

Li Jian-Xin [8] and Katarina Cechlarova [7] discussed the solvability of maxmin fuzzy equation $xA = b$ and $Ax = b$. In both the cases the maximum solution is unique and the minimum solution need not be unique. Let $\Omega_2(A, b)$ be the set of all solutions for $Ax = b$. Murugadas [12] introduced a method to find maximum g-inverse as well as minimum g-inverse of fuzzy matrix and intuitionistic fuzzy matrix. Let us restrict our further discussion in this section to fuzzy neutrosophic soft matrix equation of the form $Ax = b$ with $x = [\langle x_{i1}^T, x_{i1}^I, x_{i1}^F\rangle \mid i \in I_n], b = [\langle b_{k1}^T, b_{k1}^I, b_{k1}^F\rangle \mid k \in I_m]$ where A is a fuzzy neutrosophic soft matrix of order mn

Irfan Deli [6] introduced the new concept of npn-soft sets theory. Tanushree Mitra Basu and Shyamal Kumar Mondal[17] used neutrosophic soft matrix in group decision making problems. Said Broumi et al., [15] introduced generalized interval neutrosophic soft set.

In this paper the authors extend the idea of finding g-inverse to FNSM. And also finds the maximum and minimum solution of the relational equation $xA = b$ when A is a FNSM. Further this concept has been extended in finding g-inverse of FNSM.

2. Preliminaries

In this section, some relevant basic definitions about fuzzy neutrosophic soft matrices are provided.

Definition 2.1 [16]A neutrosophic set A on the universe of discourse X is defined as

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\},$$

where $T, I, F : X \to \ ]^-0, 1^+[$ and

$$^-0 \leqslant T_A(x) + I_A(x) + F_A(x) \leqslant 3^+. \tag{1}$$

From philosophical point of view the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. But in real life application especially in scientific and Engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0, 1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of $[0,1]$. Therefore we can rewrite the equation (1) as

$$0 \leqslant T_A(x) + I_A(x) + F_A(x) \leqslant 3.$$

In short an element $\tilde{a}$ in the neutrosophic set A, can be written as

$$\tilde{a} = \langle a^T, a^I, a^F \rangle,$$

where a^T denotes degree of truth, a^I denotes degree of indeterminacy, a^F denotes degree of falsity such that $0 \leqslant a^T + a^I + a^F \leqslant 3$.

***Example* 2.2** Assume that the universe of discourse $X = \{x_1, x_2, x_3\}$, where x_1, x_2, and x_3 characterize the quality, relaibility, and the price of the objects. It may be further assumed that the values of $\{x_1, x_2, x_3\}$ are in $[0,1]$ and they are obtained from some investigations of some experts. The experts may impose their opinion in three components viz; the degree of goodness, the degree of indeterminacy and the degree of poorness to explain the characteristics of the objects. Suppose A is a Neutrosophic Set (NS) of X, such that $A = \{\langle x_1, 0.4, 0.5, 0.3\rangle, \langle x_2, 0.7, 0.2, 0.4\rangle, \langle x_3, 0.8, 0.3, 0.4\rangle\}$, where for x_1 the degree of goodness of quality is 0.4 , degree of indeterminacy of quality is 0.5 and degree of falsity of quality is 0.3 etc,.

Definition 2.3 [11] Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a nonempty set A, $A \subset E$. A pair (F,A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

Definition 2.4 [1] Let U be an initial universe set and E be a set of parameters. Consider a non empty set A, $A \subset E$. Let $P(U)$ denotes the set of all fuzzy neutrosophic sets of U. The collection (F, A) is termed to be the Fuzzy Neutrosophic Soft Set (FNSS) over U, where F is a mapping given by $F : A \to P(U)$. Hereafter we simply consider A as FNSS over U instead of (F, A).

Definition 2.5 [2] Let $U = \{c_1, c_2, ...c_m\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, ...e_n\}$. Let $A \subseteq E$. A pair (F, A) be a FNSS over U. Then the subset of $U \times E$ is defined by $R_A = \{(u, e);\ e \in A,\ u \in F_A(e)\}$ which is called a relation form of (F_A, E). The membership function, indeterminacy membership function and non membership function are written by $T_{R_A} : U \times E \to [0,1]$, $I_{R_A} : U \times E \to [0,1]$ and $F_{R_A} : U \times E \to [0,1]$ where $T_{R_A}(u,e) \in [0,1], I_{R_A}(u,e) \in [0,1]$ and $F_{R_A}(u,e) \in [0,1]$ are the membership value, indeterminacy value and non membership value respectively of $u \in U$ for each $e \in E$. If $[(T_{ij}, I_{ij}, F_{ij})] = [(T_{ij}(u_i, e_j), I_{ij}(u_i, e_j), F_{ij}(u_i, e_j)]$ we define a matrix

$$[\langle T_{ij}, I_{ij}, F_{ij}\rangle]_{m\times n} = \begin{bmatrix} \langle T_{11}, I_{11}, F_{11}\rangle & \langle T_{12}, I_{12}, F_{12}\rangle & \cdots & \langle T_{1n}, I_{1n}, F_{1n}\rangle \\ \langle T_{21}, I_{21}, F_{21}\rangle & \langle T_{22}, I_{22}, F_{22}\rangle & \cdots & \langle T_{2n}, I_{2n}, F_{2n}\rangle \\ \vdots & \vdots & & \vdots \\ \langle T_{m1}, I_{m1}, F_{m1}\rangle & \langle T_{m2}, I_{m2}, F_{m2}\rangle & \cdots & \langle T_{mn}, I_{mn}, F_{mn}\rangle \end{bmatrix}$$

which is called an $m \times n$ FNSM of the FNSS (F_A, E) over U.

Definition 2.6 Let $U = \{c_1, c_2...c_m\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, ...e_n\}$. Let $A \subseteq E$. A pair (F, A) be a fuzzy neutrosophic soft set. Then fuzzy neutrosophic soft set (F, A) in a matrix form as $A_{m\times n} = (a_{ij})_{m\times n}$ or

$A = (a_{ij}), i = 1, 2, ...m, j = 1, 2, ...n$ where

$$(a_{ij}) = \begin{cases} (T(c_i, e_j), I(c_i, e_j), F(c_i, e_j)) & \text{if } \ e_j \in A \\ \langle 0, 0, 1\rangle & \text{if } \ e_j \notin A \end{cases}$$

where $T_j(c_i)$ represents the membership of c_i, $I_j(c_i)$ represents the indeterminacy of c_i and $F_j(c_i)$ represents the non-membership of c_i in the FNSS (F, A). If we replace the identity element $\langle 0, 0, 1\rangle$ by $\langle 0, 1, 1\rangle$ in the above form we get FNSM of type-II.

FNSM of Type-I [18]
Let $\mathcal{N}_{mn}$ denotes FNSM of order $m \times n$ and $\mathcal{N}_n$ denotes FNSM of order $n \times n$.

Definition 2.7 Let $A = (\left\langle a^T_{ij}, a^I_{ij}, a^F_{ij}\right\rangle), B = (\left\langle b^T_{ij}, b^I_{ij}, b^F_{ij}\right\rangle) \in \mathcal{N}_{m\times n}$ the componentwise addition and componentwise multiplication is defined as

$$A \oplus B = \Big(\sup\{a^T_{ij}, b^T_{ij}\}, \quad \sup\{a^I_{ij}, b^I_{ij}\}, \quad \inf\{a^F_{ij}, b^F_{ij}\}\Big),$$
$$A \odot B = \Big(\inf\{a^T_{ij}, b^T_{ij}\}, \quad \inf\{a^I_{ij}, b^I_{ij}\}, \quad \sup\{a^F_{ij}, b^F_{ij}\}\Big).$$

Definition 2.8 Let $A \in \mathcal{N}_{mn}, B \in \mathcal{N}_{np}$, the composition of A and B is defined as

$$A \circ B = \left(\sum_{k=1}^{n}(a^T_{ik} \wedge \ b^T_{kj}), \quad \sum_{k=1}^{n}(a^I_{ik} \wedge \ b^I_{kj}), \quad \prod_{k=1}^{n}(a^F_{ik} \vee b^F_{kj})\right)$$

equivalently we can write the same as

$$A \circ B = \left(\bigvee_{k=1}^{n}(a^T_{ik} \wedge b^T_{kj}), \quad \bigvee_{k=1}^{n}(a^I_{ik} \wedge b^I_{kj}), \quad \bigwedge_{k=1}^{n}(a^F_{ik} \vee b^F_{kj})\right).$$

The product $A \circ B$ is defined if and only if the number of columns of A is same as the number of rows of B. A and B are said to be conformable for multiplication. We shall use AB instead of $A \circ B$.

FNSM of Type-II[18]

Definition 2.9 Let $A = (\langle a^T_{ij}, a^I_{ij}, a^F_{ij}\rangle), B = (\langle b^T_{ij}, b^I_{ij}, b^F_{ij}\rangle) \in \mathcal{N}_{m\times n}$, the component wise addition and component wise multiplication is defined as

$$A \oplus B = \Big(\Big\langle \sup\{a^T_{ij}, b^T_{ij}\}, \quad \inf\{a^I_{ij}, b^I_{ij}\}, \quad \inf\{a^F_{ij}, b^F_{ij}\}\Big\rangle\Big),$$
$$A \odot B = \Big(\Big\langle \inf\{a^T_{ij}, b^T_{ij}\}, \quad \sup\{a^I_{ij}, b^I_{ij}\}, \quad \sup\{a^F_{ij}, b^F_{ij}\}\Big\rangle\Big).$$

Analogous to FNSM of type-I, we can define FNSM of type -II in the following way

Definition 2.10 Let

$$A = (\langle a^T_{ij}, a^I_{ij}, a^F_{ij}\rangle) = (a_{ij}) \in \mathcal{N}_{m\times n} \quad \text{and} \quad B = (\langle b^T_{ij}, b^I_{ij}, b^F_{ij}\rangle) = (b_{ij}) \in \mathcal{F}_{n\times p},$$

the product of A and B is defined as

$$A * B = \left(\sum_{k=1}^{n} \langle a_{ik}^T \wedge \ b_{kj}^T \rangle , \quad \prod_{k=1}^{n} \langle a_{ik}^I \vee \ b_{kj}^I \rangle , \quad \prod_{k=1}^{n} \langle a_{ik}^F \vee b_{kj}^F \rangle \right)$$

equivalently we can write the same as

$$A * B = \left(\bigvee_{k=1}^{n} \langle a_{ik}^T \wedge b_{kj}^T \rangle , \bigwedge_{k=1}^{n} \langle a_{ik}^I \vee b_{kj}^I \rangle , \bigwedge_{k=1}^{n} \langle a_{ik}^F \vee b_{kj}^F \rangle \right).$$

the product $A * B$ is defined if and only if the number of columns of A is same as the number of rows of B. A and B are said to be conformable for multiplication.

3. Main results

Definition 3.1 $A \in N_{m\times n}$ is said to be regular if there exists $X \in N_{n\times m}$ such that $AXA = A$.

Definition 3.2 If A and X are two FNSM of order $m\times n$ satisfies the relation $AXA = A$, then X is called a generalized inverse (g-inverse) of A which is denoted by A^-. The g-inverse of an FNSM is not necessarily unique. We denote the set of all g-inverse by $A\{1\}$.

Definition 3.3 Any element $\hat{x} \in \Omega_1(A, b)$ is called a maximal solution if

$$\forall\, x \in \Omega_1(A, b), \;\; x \geqslant \hat{x} \;\Rightarrow\; x = \hat{x}.$$

That is elements $x, \hat{x}$ are component wise equal.

Definition 3.4 Any element $\check{x} \in \Omega_1(A, b)$ is called a minimal solution if

$$\forall\, x \in \Omega_1(A, b), \;\; x \leqslant \check{x} \;\Rightarrow\; x = \check{x}.$$

That is elements $x, \check{x}$ are component wise equal.

Lemma 3.5 Let $xA = b$ be as defined in equation (1). If

$$\langle \max_j a_{jk}^T, \max_j a_{jk}^I, \min_j a_{jk}^F \rangle < \langle b_{1k}^T, b_{1k}^I, b_{1k}^F \rangle,$$

for some $k \in I_n$, then $\Omega_1(A, b) = \phi$.

Proof. If $\langle \max_j a_{jk}^T, \max_j a_{jk}^I, \min_j a_{jk}^F \rangle < \langle b_{1k}^T, b_{1k}^I, b_{1k}^F \rangle$ for some k, then

$$\begin{aligned} \min\{\langle x_{1j}^T, x_{1j}^I, x_{1j}^F \rangle, \langle a_{jk}^T, a_{jk}^I, a_{jk}^F \rangle\} &\leqslant \langle a_{jk}^T, a_{jk}^I, a_{jk}^F \rangle \\ &\leqslant \langle \max_j a_{jk}^T, \max_j a_{jk}^I, \min_j a_{jk}^F \rangle \\ &< \langle b_{1k}^T, b_{1k}^I, b_{1k}^F \rangle. \end{aligned}$$

Hence

$$\max_j \min\left\{\langle x^T_{1j}, x^I_{1j}, x^F_{1j}\rangle, \langle a^T_{jk}, a^I_{jk}, a^F_{jk}\rangle\right\} < \langle b^T_{1k}, b^I_{1k}, b^F_{1k}\rangle.$$

Therefore, no values of x satisfy the equation $xA = b$. Hence $\Omega(A, b) = \phi$. ■

Theorem 3.6 For the equation $xA = b$, $\Omega_1(A, b) \neq \varphi$ if and only if

$$\hat{x} = [\langle \hat{x}^T_{1j}, \hat{x}^I_{1j}, \hat{x}^F_{1j}\rangle | j \in I_m]$$

defined as

$$\langle \hat{x}^T_{1j}, \hat{x}^I_{1j}, \hat{x}^F_{1j}\rangle = \langle \min \sigma(a^T_{jk}, b^T_{1k}), \min \sigma^{'}(a^I_{jk}, b^I_{1k}), \max \sigma^{''}(a^F_{jk}, b^F_{1k})\rangle,$$

where

$$\sigma(a^T_{jk}, b^T_{1k}) = \begin{cases} b^T_{1k} & \text{if} \quad a^T_{jk} > b^T_{1k} \\ 1 & \text{otherwise} \end{cases}$$

$$\sigma'(a^I_{jk}, b^I_{jk}) = \begin{cases} b^I_{1k} & \text{if} \quad a^I_{jk} > b^I_{1k} \\ 1 & \text{otherwise} \end{cases}$$

$$\sigma''(a^F_{jk}, b^F_{1k}) = \begin{cases} b^F_{1k} & \text{if} \quad a^F_{jk} < b^F_{1k} \\ 0 & \text{otherwise} \end{cases}$$

is the maximum solution of $xA = b$.

Proof. If $\Omega_1(A, b) \neq \phi$, then $\hat{x}$ is a solution of $xA = b$. For if $\hat{x}$ is not a solution, then $\hat{x}A \neq b$ and therefore

$$\max_j \min\langle \hat{x}^T_{1j}, \hat{x}^I_{1j}, \hat{x}^F_{1j}\rangle\langle a^T_{jk}, a^I_{jk}, a^F_{jk}\rangle \neq \langle b^T_{1k_0}, b^I_{1k_0}, b^F_{1k_0}\rangle$$

for atleast one $k_0 \in I_n$. By the Definition of $\langle \hat{x}^T_{1j}, \hat{x}^I_{1j}, \hat{x}^F_{1j}\rangle$,

$$\langle \hat{x}^T_{1j}, \hat{x}^I_{1j}, \hat{x}^F_{1j}\rangle \leqslant \langle b^T_{1k}, b^I_{1k}, b^F_{1k}\rangle$$

for each k and so $\langle \hat{x}^T_{1j}, \hat{x}^I_{1j}, \hat{x}^F_{1j}\rangle \leqslant \langle b^T_{1k_0}, b^I_{1k_0}, b^F_{1k_0}\rangle$. Therefore,

$$\langle \hat{x}^T_{1j}, \hat{x}^I_{1j}, \hat{x}^F_{1j}\rangle\langle a^T_{jk}, a^I_{jk}, a^F_{jk}\rangle < \langle b^T_{1k_0}, b^I_{1k_0}, b^F_{1k_0}\rangle$$
$$\langle \max_j a^T_{jk}, \max_j a^I_{jk}, \min_j a^F_{jk}\rangle < \langle b^T_{1k_0}, b^I_{1k_0}, b^F_{1k_0}\rangle,$$

for some k_0 by our assumption. Hence by Lemma 3.5 $\Omega_1(A, b) = \phi$, which is a contradiction. Hence $\hat{x}$ is a solution. Let us prove that $\hat{x}$ is the maximum one. If possible let us assume that $\hat{y}$ is another solution such that $\hat{y} \geqslant \hat{x}$ that is

$$\langle y^T_{1j_0}, y^I_{1j_0}, y^F_{1j_0}\rangle > \langle \hat{x}^T_{1j_0}, \hat{x}^I_{1j_0}, \hat{x}^F_{1j_0}\rangle$$

for atleast one j_0. Therefore, by the definition of $\langle \hat{x}^T_{1j_0}, \hat{x}^I_{1j_0}, \hat{x}^F_{1j_0} \rangle$,

$$\langle y^T_{1j_0}, y^I_{1j_0}, y^F_{1j_0} \rangle > \langle \min \sigma(a^T_{j_0k}, b^T_{1k}), \min \sigma^{'}(a^I_{j_0k}, b^I_{1k}), \max \sigma^{''}(a^F_{j_0k}, b^F_{1k}) \rangle.$$

Since $\Omega(A, b) \neq \phi$, by the Lemma 3.5 $\langle \max\limits_j a^T_{jk}, \max\limits_j a^I_{jk}, \min\limits_j a^F_{jk} \rangle \geqslant \langle b^T_{1k_0}, b^I_{1k_0}, b^F_{1k_0} \rangle$ for each k_0. Hence

$$\langle b^T_{1k_0}, b^I_{1k_0}, b^F_{1k_0} \rangle \neq \langle \max_j \min(y^T_j, a^T_{jk_0}), \max_j min(y^I_j, a^I_{jk_0}), \min_j \max(y^F_j, a^F_{jk_0}) \rangle$$

which is a contradiction to our assumption that $y \in \Omega_1(A, b)$. Therefore $\hat{x}$ is the maximum solution. The converse part is trivial. If the relational equation is the form $Ax = b \cdots (2)$ where A is an fuzzy neutrosophic soft matrix of order $m \times n$,

$$x = (\langle x^T_{11}, x^I_{11}, x^F_{11} \rangle, ..., \langle x^T_{1n}, x^I_{1n}, x^F_{1n} \rangle)^T,$$
$$b = (\langle b^T_{11}, b^I_{11}, b^F_{11} \rangle, ..., \langle b^T_{1m}, b^I_{1m}, b^F_{1m} \rangle)^T$$

we can prove the following Lemma and Theorem in similar fashion. Let $\Omega_2(A, b)$ be the set all solution of the relational equation $Ax = b$. ■

Definition 3.7 Any element $\hat{x} \in \Omega_2(A, b)$ is called a maximal solution if

$$\forall\, x \in \Omega_2(A, b), \;\; x \geqslant \hat{x} \Rightarrow \; x = \hat{x}.$$

That is elements $x, \hat{x}$ are component wise equal.

Definition 3.8 Any element $\check{x} \in \Omega_2(A, b)$ is called a minimal solution if

$$\forall\, x \in \Omega_2(A, b), \;\; x \leqslant \check{x} \Rightarrow \; x = \check{x}.$$

That is elements $x, \check{x}$ are component wise equal.

Lemma 3.9 Let $Ax = b$ as defined in (2). If

$$\langle \max_i a^T_{ki}, \max_i a^I_{ki}, \min_i a^F_{ki} \rangle < \langle b^T_{k1}, b^I_{k1}, b^F_{k1} \rangle,$$

for some $k \in I_m$, then $\Omega_2(A, b) = \phi$.

Theorem 3.10 For the equation $Ax = b$, $\Omega_2(A, b) \neq \phi$ if only if

$$\hat{x} = [\langle \hat{x}^T_{j1}, \hat{x}^I_{j1}, \hat{x}^F_{j1} \rangle | j \in I_n]$$

defined as

$$\langle \hat{x}^T_{j1}, \hat{x}^I_{j1}, \hat{x}^F_{j1} \rangle = \langle \min \sigma(a^T_{ki}, b^T_{k1}), \min \sigma'(a^I_{ki}, b^I_{k1}), \max \sigma''(a^F_{ki}, b^F_{k1}) \rangle,$$

where

$$\sigma(a^T_{ki}, b^T_{k1}) = \begin{cases} b^T_{k1} & \text{if} \quad a^T_{ki} > b^T_{k1} \\ 1 & \text{otherwise} \end{cases}$$

$$\sigma'(a^I_{ki}, b^I_{k1}) = \begin{cases} b^I_{k1} & \text{if} \quad a^I_{ki} > b^I_{k1} \\ 1 & \text{otherwise} \end{cases}$$

$$\sigma''(a^F_{ki}, b^F_{k1}) = \begin{cases} b^F_{1k} & \text{if} \quad a^F_{ki} < b^F_{k1} \\ 0 & \text{otherwise} \end{cases}$$

is the maximum solution of $Ax = b$.

***Example* 3.11** Let

$$A = \begin{bmatrix} \langle 0.5\ 0.6\ 0.2\rangle & \langle 0.7, 0.5, 0.1\rangle \\ \langle 0.2\ 0.3\ \ 0.5\rangle & \langle 0.6, 0.4, 0\rangle \end{bmatrix} \quad \text{and} \quad b = [\langle 0.2, 0.3, 0.5\rangle \quad \langle 0.5, 0.3, 0.1\rangle].$$

Then we can find $\hat{x} = [\langle \hat{x}^T_{11}, \hat{x}^I_{11}, \hat{x}^F_{11}\rangle, \langle \hat{x}^T_{12}, \hat{x}^I_{12}, \hat{x}^F_{12}\rangle]$ in $xA = b$

$$\begin{aligned} \langle \hat{x}^T_{11}, \hat{x}^I_{11}, \hat{x}^F_{11}\rangle &= \langle \min_k \sigma(a^T_{1k}, b^T_{1k}), \min_k \sigma^{'}(a^I_{1k}, b^I_{1k}), \max_k \sigma^{''}(a^F_{1k}, b^F_{1k})\rangle \\ &= \langle \min_k(0.2, 0.5), \min_k(0.3, 0.3), \max_k(0.5, 0)\rangle \\ &= \langle 0.2, 0.3, 0.5\rangle \\ \langle \hat{x}^T_{12}, \hat{x}^I_{12}, \hat{x}^F_{12}\rangle &= \langle \min_k \sigma(a^T_{2k}, b^T_{1k}), \min_k \sigma^{'}(a^I_{2k}, b^I_{1k}), \max_k \sigma^{''}(a^F_{2k}, b^F_{1k})\rangle \\ &= \langle \min_k(1, 0.5), \min_k(1, 0.3), \max_k(0, 0.1)\rangle \\ &= \langle 0.5, 0.3, 0.1\rangle \end{aligned}$$

Then clearly

$$(\langle 0.2, 0.3, 0.5\rangle\langle 0.5, 0.3, 0.1\rangle) \begin{bmatrix} \langle 0.5\ 0.6\ 0.2\rangle\langle 0.7, 0.5, 0.1\rangle \\ \langle 0.2\ 0.3\ \ 0.5\rangle\langle 0.6, 0.4, 0\rangle \end{bmatrix} = (\langle 0.2, 0.3, 0.5\rangle \quad \langle 0.5, 0.3, 0.1\rangle)$$

To get the minimal solution of $xA = b$ we follow the procedure as followed for fuzzy neutrosophic soft matrix equation.

Step 1 Determine the sets

$$J_k(\hat{x}) = \Big\{ j \in I_m \mid \min(\langle \hat{x}^T_{1j}, \hat{x}^I_{1j}, \hat{x}^F_{1j}\rangle, \langle a^T_{jk}, a^I_{jk}, a^F_{jk}\rangle) = b^T_{1k} \Big\},$$

for all $k \in I_n$. Construct their cartesian product $J(\hat{x}) = J_1(\hat{x}) \times J_2(\hat{x}) \times ... \times J_n(\hat{x})$.

Step 2 Denote the elements of $J(\hat{x})$, by $\beta = [\beta_k/k \in I_n]$. For each $\beta \in J(\hat{x})$ and each $j \in I_m$, determine the set

$$k(\beta, j) = \Big\{ k \in I_m \mid \beta_k = j \Big\}.$$

Step 3 For each $\beta \in J(\hat{x})$ generate the n-tuple

$$g(\beta) = \Big\{ g_j(\beta) \mid j \in I_m \Big\},$$

where

$$g_j(\beta) = \begin{cases} \max\limits_{k(\beta,j)} \langle b^T_{1k}, b^I_{1k}, b^F_{1k} \rangle & k(\beta, j) \neq 0 \\ \langle 0, 0, 1 \rangle & \text{otherwise} \end{cases}$$

Step 4 From all the n-tuples $g(\beta)$ generated in step.3, select only the minimal one by pairwise comparison. The resulting set of n-tuples is the minimal solution of the reduced form of equation $xA = b$.

***Example* 3.12** Let us find the minimal solution to the linear equation given in Example 3.11 using the maximal solution $\hat{x}$

Step 1 To determine $J_k(\hat{x})$ for $k = 1, 2$.

$$\begin{aligned} J_1(\hat{x}) &= \{j = 1, 2 | \min(\langle x^T_{1j}, x^I_{1j}, x^F_{1j} \rangle, \langle a^T_{jk}, a^I_{jk}, a^F_{jk} \rangle) = \langle b^T_{1k}, b^I_{1k}, b^F_{1k} \rangle\} \\ &= \{\min\{\langle 0.2, 0.3, 0.5 \rangle \langle 0.5, 0.6, 0.2 \rangle\}, \min\{\langle 0.5, 0.3, 0.1 \rangle \langle 0.2, 0.3, 0.5 \rangle\}\} \\ &= \langle 0.2, 0.3, 0.5 \rangle = \{1, 2\} \\ J_2(\hat{x}) &= \{\min\{\langle 0.2, 0.3, 0.5 \rangle \langle 0.7, 0.5, 0.1 \rangle\}, \min\{\langle 0.5, 0.3, 0.1 \rangle \langle 0.6, 0.4, 0 \rangle\}\} \\ &= \langle 0.5, 0.3, 0.1 \rangle \\ &= \{2\} \end{aligned}$$

Therefore

$$J_k(\hat{x}) = J_1(\hat{x}) \times J_2(\hat{x}) = \{1, 2\} \times \{2\} = \{(1, 2, (2, 2))\} = \beta$$

Step 2 To determine the sets $K(\beta, j)$ for each $\beta = J_k(\hat{x})$ and for each $j = 1, 2$.
For $\beta = (1, 2)$

$$\begin{aligned} K(\beta, 1) &= \{k = 1, 2 | \beta_k = 1\} = \{1\} \\ K(\beta, 2) &= \{k = 1, 2 | \beta_k = 2\} = \{2\} \end{aligned}$$

For $\beta = (2, 2)$

$$\begin{aligned} K(\beta, 1) &= \{k = 1, 2 | \beta_k = 1\} = \{\varphi\} \\ K(\beta, 2) &= \{k = 1, 2 | \beta_k = 2\} = \{1, 2\} \end{aligned}$$

Thus the sets $K(\beta, j)$ for each $\beta \in J(\hat{x})$ and $j = 1, 2$ are listed in the following table.

$K\{\beta, j\}$	1	K
$(1, 2)$	$\{1\}$	$\{2\}$
$(2, 2)$	$\{\phi\}$	$\{1, 2\}$

Step 3 For each $\beta \in J(\hat{x})$ we generate the tuples $g(\beta)$
For $\beta = (1, 2)$

$$g_1(\beta) = \begin{cases} \max\limits_{k \in k(\beta,1)} \langle 0.2, 0.3, 0.5\rangle & k(\beta, 1) \neq \phi \\ \langle 0, 0, 1\rangle & \text{otherwise} \end{cases} = \langle 0.2, 0.3, 0.5\rangle$$

$$g_2(\beta) = \langle 0.5, 0.3, 0.1\rangle.$$

For $\beta = (2, 2)$

$$g_1(\beta) = \langle 0, 0, 1\rangle$$
$$g_2(\beta) = \langle 0.5, 0.3, 0.1\rangle.$$

Therefore we can get the following table for β

β	$g(\beta)$
$(1, 2)$	$\langle 0.2, 0.3, 0.5\rangle, \langle 0.5, 0.3, 0.1\rangle$
$(2, 2)$	$\langle 0, 0, 1\rangle, \langle 0.5, 0.3, 0.1\rangle$

Out of which $(\langle 0, 0, 1\rangle, \langle 0.5, 0.3, 0.1\rangle)$ is the minimal one. And also it satisfy $xA = b$ that is $\hat{x} = (\langle 0, 0, 1\rangle, \langle 0.5, 0.3, 0.1\rangle)$. Using the same method we have followed, one can find the g-inverse of a fuzzy neutrosophic soft matrix if it exits.

***Example* 3.13** Let $A = \begin{bmatrix} \langle 1,1,0\rangle & \langle 1,1,0\rangle \\ \langle 1,1,0\rangle & \langle 0,0,1\rangle \end{bmatrix}$. To find the g-inverse, put $AXA = A$ and $AX = B$ so that $BA = A$, where

$$X = \begin{bmatrix} \langle x_{11}^T, x_{11}^I, x_{11}^F\rangle & \langle x_{12}^T, x_{12}^I, x_{12}^F\rangle \\ \langle x_{21}^T, x_{21}^I, x_{21}^F\rangle & \langle x_{22}^T, x_{22}^I, x_{22}^F\rangle \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} \langle b_{11}^T, b_{11}^I, b_{11}^F\rangle & \langle b_{12}^T, b_{12}^I, b_{12}^F\rangle \\ \langle b_{21}^T, b_{21}^I, b_{21}^F\rangle & \langle b_{22}^T, b_{22}^I, b_{22}^F\rangle \end{bmatrix}.$$

To find B and X:

$$\big(\langle b_{11}^T, b_{11}^I, b_{11}^F\rangle, \langle b_{12}^T, b_{12}^I, b_{12}^F\rangle\big) \begin{bmatrix} \langle 1,1,0\rangle & \langle 1,1,0\rangle \\ \langle 1,1,0\rangle & \langle 0,0,1\rangle \end{bmatrix} = \big(\langle 1,1,0\rangle, \langle 1,1,0\rangle\big)$$

$$\langle b_{11}^T, b_{11}^I, b_{11}^F\rangle = \Big\langle \min_k \sigma(a_{1k}, b_{1k}), \min_k \sigma'(a'_{1k}, b'_{1k}), \max_k \sigma''(a''_{1k}, b''_{1k})\Big\rangle = \langle 1,1,0\rangle$$

$$\langle b_{12}^T, b_{12}^I, b_{12}^F\rangle = \langle \min_k \sigma(a_{2k}, b_{2k}), \min_k \sigma'(a'_{2k}, b'_{2k}), \max_k \sigma''(a''_{2k}, b''_{2k})\rangle = \langle 1,1,0\rangle$$

Take

$$\big(\langle b_{21}^T, b_{21}^I, b_{21}^F\rangle, \langle b_{22}^T, b_{22}^I, b_{22}^F\rangle\big) \begin{bmatrix} \langle 1,1,0\rangle & \langle 1,1,0\rangle \\ \langle 1,1,0\rangle & \langle 0,0,1\rangle \end{bmatrix} = \big(\langle 1,1,0\rangle, \langle 0,0,1\rangle\big)$$

$$\langle b_{21}^T, b_{21}^I, b_{21}^F\rangle = \langle \min_k \sigma_{1k}, b_{2k}), \min_k(\sigma'_{1k}, b'_{2k}), \max_k(\sigma''_{1k}, b''_{2k})\rangle = \langle 0,0,1\rangle$$

$$\langle b_{22}^T, b_{22}^I, b_{22}^F\rangle = \langle \min_k(\sigma_{2k}, b_{2k}), \min_k(\sigma'_{2k}, b'_{2k}), \max_k(\sigma''_{2k}, b''_{2k})\rangle = \langle 1,1,0\rangle$$

Therefore $\hat{B} = \begin{bmatrix} \langle 1,1,0\rangle & \langle 1,1,0\rangle \\ \langle 0,0,1\rangle & \langle 1,1,0\rangle \end{bmatrix}$ which satisfy $BA = A$. Moreover, $AX = B$ becomes

$$\begin{bmatrix} \langle 1,1,0\rangle & \langle 1,1,0\rangle \\ \langle 0,0,1\rangle & \langle 1,1,0\rangle \end{bmatrix} \begin{bmatrix} \langle x_{11}^T, x_{11}^I, x_{11}^F\rangle & \langle x_{12}^T, x_{12}^I, x_{12}^F\rangle \\ \langle x_{21}^T, x_{21}^I, x_{21}^F\rangle & \langle x_{22}^T, x_{22}^I, x_{22}^F\rangle \end{bmatrix} = \begin{bmatrix} \langle 1,1,0\rangle & \langle 1,1,0\rangle \\ \langle 0,0,1\rangle & \langle 1,1,0\rangle \end{bmatrix}$$

$$\langle x_{11}^T, x_{11}^I, x_{11}^F\rangle = \langle \min_k(\sigma_{k1}, b_{k1}), \min_k(\sigma'_{k1}, b'_{k1}), \max_k(\sigma''_{k1}, b''_{k1})\rangle = \langle 0,0,1\rangle$$

$$\langle x_{12}^T, x_{12}^I, x_{12}^F\rangle = \langle \min_k(\sigma_{k1}, b_{k2}), \min_k(\sigma'_{k1}, b'_{k2}), \max_k(\sigma''_{k1}, b''_{k2})\rangle = \langle 1,1,0\rangle$$

$$\langle x_{21}^T, x_{21}^I, x_{21}^F\rangle = \langle \min_k(\sigma_{k2}, b_{k1}), \min_k(\sigma'_{k2}, b'_{k1}), \max_k(\sigma''_{k2}, b''_{k1})\rangle = \langle 1,1,0\rangle$$

$$\langle x_{22}^T, x_{22}^I, x_{22}^F\rangle = \langle \min_k(\sigma_{k2}, b_{k2}), \min_k(\sigma'_{k2}, b'_{k2}), \max_k(\sigma''_{k2}, b''_{k2})\rangle = \langle 1,1,0\rangle$$

Therefore

$$\hat{X} = \begin{bmatrix} \langle 0,0,1\rangle & \langle 1,1,0\rangle \\ \langle 1,1,0\rangle & \langle 1,1,0\rangle \end{bmatrix}.$$

Clearly $A\hat{X}A = A$. Hence $\hat{X}$ is the maximum g-inverse of $A\hat{X}A = A$. To get the minimal solution: Let us find the minimum $\hat{B}$ from $\hat{B}A = A$ and using the minimum $\hat{B}$ in $AX = B$ we can find the minimum $\hat{X}$. Consider

$$\begin{bmatrix} \langle 1,1,0\rangle & \langle 1,1,0\rangle \\ \langle 0,0,1\rangle & \langle 1,1,0\rangle \end{bmatrix} \begin{bmatrix} \langle 1,1,0\rangle & \langle 1,1,0\rangle \\ \langle 1,1,0\rangle & \langle 0,0,1\rangle \end{bmatrix} = \begin{bmatrix} \langle 1,1,0\rangle & \langle 1,1,0\rangle \\ \langle 1,1,0\rangle & \langle 0,0,1\rangle \end{bmatrix}.$$

Step 1 Determine the set $J_{ij}(\hat{B})$

$$\begin{aligned} J_{11}(\hat{B}) &= \{min\{\langle 1,1,0\rangle, \langle 1,1,0\rangle\}, min\{\langle 1,1,0\rangle, \langle 1,1,0\rangle\}\} = \langle 1,1,0\rangle \\ &= \{\langle 1,1,0\rangle, \langle 1,1,0\rangle\} = \{1,2\} \\ J_{12}(\hat{B}) &= \{min\{\langle 1,1,0\rangle, \langle 1,1,0\rangle\}, min\{\langle 1,1,0\rangle, \langle 0,0,1\rangle\}\} = \langle 1,1,0\rangle \\ &= \{\langle 1,1,0\rangle, \langle 0,0,1\rangle\} = \{1\} \\ J_{21}(\hat{B}) &= \{min\{\langle 0,0,1\rangle, \langle 1,1,0\rangle\}, min\{\langle 1,1,0\rangle, \langle 1,1,0\rangle\}\} = \langle 1,1,0\rangle \\ &= \{\langle 0,0,1\rangle, \langle 1,1,0\rangle\} = \{2\} \\ J_{22}(\hat{B}) &= \{min\{\langle 0,0,1\rangle, \langle 1,1,0\rangle\}, min\{\langle 1,1,0\rangle, \langle 0,0,1\rangle\}\} = \langle 0,0,1\rangle \\ &= \{\langle 0,0,1\rangle, \langle 0,0,1\rangle\} = \{1,2\} \end{aligned}$$

Let $\beta_1 = J_{11}(\hat{B}) \times J_{12}(\hat{B}) = \{1,2\} \times \{1\} = \{(1,2),(2,1)\}$
$\beta_2 = J_{21}(\hat{B}) \times J_{22}(\hat{B}) = \{2\} \times \{1,2\} = \{(2,1),(2,2)\}$
Step 2. Determine the set $K(\beta_k, j)$ for $k = 1,2$ and $j = 1,2$
For $\beta_1 = (1,1) K(\beta_1, 1) = \{1,2\}$
$K(\beta_1, 2) = \{\phi\}$
For $\beta_1 = (2,1) K(\beta_1, 1) = \{2\}$
$K(\beta_1, 2) = \{1\}$
For $\beta_2 = (2,1) K(\beta_2, 1) = \{2\}$
$K(\beta_2, 2) = \{1\}$

For $\beta_2 = (2,2) K(\beta_2, 1) = \{\phi\}$
$K(\beta_2, 2) = \{1, 2\}$
Writing the values in tabular form we get

(β_1, j)	1	2
$(1,1)$	$\{1,2\}$	ϕ
$(2,1)$	$\{2\}$	1

(β_2, j)	1	2
$(2,1)$	$\{2\}$	$\{1\}$
$(2,2)$	$\{\phi\}$	$\{1,2\}$

Step 3. For each β_k let us generate the $g(\beta_k)$ tuples
For $\beta_1 = (1,1)$
$g_1(\beta_1) = (1,1)$
$g_1(\beta_1) = \max_{k \in K(\beta,1)} \{\langle 1,1,0\rangle, \langle 1,1,0\rangle\} = \langle 1,1,0\rangle$
$g_2(\beta_1) = \langle 0,0,1\rangle$
For $(\beta_1) = (2,1)$
$g_1(\beta_1) = \langle 1,1,0\rangle$
$g_2(\beta_1) = \langle 1,1,0\rangle$
For $g_1(\beta_1) = (2,1)$
$g_1(\beta_1) = \langle 1,1,0\rangle$
$g_2(\beta_1) = \langle 1,1,0\rangle$
For $g_1(\beta_1) = (2,2)$
$g_1(\beta_2) = \langle 0,0,1\rangle$
$g_2(\beta_2) = max\{\langle 1,1,0\rangle, \langle 0,0,1\rangle\} = \langle 1,1,0\rangle$
The corresponding tabular forms are given by

(β_1, j)	$g(\beta_1)$
$(1,1)$	$(\langle 1,1,0\rangle, \langle 0,0,1\rangle)$
$(2,1)$	$(\langle 1,1,0\rangle, \langle 1,1,0\rangle)$

(β_2, j)	$g(\beta_2)$
$(2,1)$	$(\langle 0,0,1\rangle, \langle 1,1,0\rangle)$
$(2,1)$	$(\langle 0,0,1\rangle, \langle 1,1,0\rangle)$

By pairwise comparison we can find out the minimum in each of the above table, we get

$$\check{B} = \begin{bmatrix} \langle 1,1,0\rangle & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & \langle 1,1,0\rangle \end{bmatrix}$$

Using the minimum $\check{B}$ in $AX = B$ we can find the minimum $\check{X}$. Now $AX = \check{B}$ is

$$\begin{bmatrix} \langle 1,1,0\rangle & \langle 1,1,0\rangle \\ \langle 1,1,0\rangle & \langle 0,0,1\rangle \end{bmatrix} \begin{bmatrix} \langle 0,0,1\rangle & \langle 1,1,0\rangle \\ \langle 1,1,0\rangle & \langle 1,1,0\rangle \end{bmatrix} = \begin{bmatrix} \langle 1,1,0\rangle & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & \langle 1,1,0\rangle \end{bmatrix}$$

Step. 4 Determine the set $J_{ij}(\hat{B})$

$$
\begin{aligned}
J_{11}(\hat{X}) &= \{\min\langle\{\langle 1,1,0\rangle, \langle 0,0,1\rangle\}, \min\langle\{\langle 1,1,0\rangle, \langle 1,1,0\rangle\}\} = \langle 1,1,0\rangle \\
&= \{\langle 0,0,1\rangle\langle 1,1,0\rangle = \{2\} \\
J_{12}(\hat{X}) &= \{\min\langle\{\langle 1,1,0\rangle, \langle 1,1,0\rangle\}, \min\langle\{\langle 1,1,0\rangle, \langle 1,1,0\rangle\}\} = \langle 0,0,1\rangle \\
&= \{\langle 1,1,0\rangle\langle 1,1,0\rangle = \{\phi\} \\
J_{21}(\hat{X}) &= \{\min\langle\{\langle 1,1,0\rangle, \langle 0,0,1\rangle\}, \min\langle\{\langle 0,0,1\rangle, \langle 1,1,0\rangle\}\} = \langle 0,0,1\rangle \\
&= \{\langle 0,0,1\rangle\langle 0,0,1\rangle = \{1,2\} \\
J_{22}(\hat{X}) &= \{\min\langle\{\langle 1,1,0\rangle, \langle 1,1,0\rangle\}, \min\langle\{\langle 0,0,1\rangle, \langle 1,1,0\rangle\}\} = \langle 1,1,0\rangle \\
&= \{\langle 1,1,0\rangle\langle 0,0,1\rangle = \{1\}
\end{aligned}
$$

Let $\beta_1 = J_{11}\hat{B} \times J_{12}\hat{B} = \{2\} \times \phi$
$\beta_2 = J_{21}\hat{B} \times J_{22}\hat{B} = \{1,2\} \times \{1\} = \{(1,1)(2,1)\}$
Step 5. Determine the set $K(\beta_k, j)$ for k=1,2 and j=1,2
For $\beta_1 = \{2\}$, $K(\beta_1, 1) = \phi$ and $K(\beta_1, 2) = \{1\}$
For $\beta_2 = \{2,1\}$, $K(\beta_2, 1) = \{2\}$ and $K(\beta_2, 2) = \{1\}$

$k(\beta_1, j)$	1	2
$\{2\} \times \phi$	ϕ	$\{1\}$

$k(\beta_2, j)$	$\{1,2\}$	ϕ
$(1,1)$	$\{1,2\}$	ϕ
$(2,1)$	$\{2\}$	$\{1\}$

Step 6: For each β_k Let as generate the $g(\beta_k)$ tuples:
For $\beta_1 = \{2\} \times \phi$ put $g_1(\beta_1) = \langle 0,0,1\rangle$ and $g_2(\beta_1) = \langle 1,1,0\rangle$
For $\beta_2 = (1,1)$ put $g_1(\beta_2) = \langle 1,1,0\rangle$ and $g_2(\beta_2) = \langle 0,0,1\rangle$
For $\beta_2 = (2,1)$ put $g_1(\beta_2) = \langle 1,1,0\rangle$ and $g_2(\beta_2) = \langle 0,0,1\rangle$
The corresponding tabular forms are given by

β_1	$g(\beta_1)$
$\{2\} \times \phi$	$\langle 0,0,1\rangle, \langle 1,1,0\rangle$

β_2	$g(\beta_2)$
$(1,1)$	$\langle 1,1,0\rangle, \langle 0,0,1\rangle$
$(2,1)$	$\langle 1,1,0\rangle, \langle 0,0,1\rangle$

To get the $\check{X}$ select a minimum row from each table, that is

$$\check{X} = \begin{bmatrix} \langle 0,0,1\rangle & \langle 1,1,0\rangle \\ \langle 1,1,0\rangle & \langle 0,0,1\rangle \end{bmatrix}$$

Clearly this $\check{X}$ will satisfy $AXA = A$ and we observe that

$$[\check{X}, \hat{X}] = \left\{ \begin{bmatrix} \langle 0,0,1\rangle & \langle 1,1,0\rangle \\ \langle 1,1,0\rangle & \langle \alpha, \alpha^{'}, \alpha^{''}\rangle \end{bmatrix} \mid 0 \leqslant \alpha \leqslant 1,\ 0 \leqslant \alpha^{'} \leqslant 1,\ \ 0 \leqslant \alpha^{''} \leqslant 1,\ \alpha + \alpha^{'} + \alpha^{''} \leqslant 3 \right\}$$

is the set of all g-inverse in $[\check{X}, \hat{X}]$.

4. Conclusion

The maximum and minimum solution of the relational equations $xA = b$ and $Ax = b$ has been obtained. Using these relational equations maximum and minimum g-inverse of a fuzzy neutrosophic soft matrix are also found. Using this g-inverse concept, further work is planned to find the necessary and sufficient condition for the existence of unique minimal solution of these relational equations $xA = b$ and $Ax = b$ for fuzzy neutrosophic soft matrices.

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