

## Fuzzy almost generalized $e$ -continuous mappings

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**Abstract.** In this paper, we introduce and characterize the concept of fuzzy almost generalized  $e$ -continuous mappings. Several interesting properties of these mappings are also given. Examples and counter examples are also given to illustrate the concepts introduced in the paper. We also introduce the concept of fuzzy  $fT_{\frac{1}{2}}$ - $e$ -space, fuzzy  $ge$ -space, fuzzy regular  $ge$ -space and fuzzy generalized  $e$ -compact space. It is seen that a fuzzy almost generalized  $e$ -continuous mapping from a fuzzy  $fT_{\frac{1}{2}}$ - $e$ -space to another fuzzy topological space becomes fuzzy almost continuous mapping.

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## 1. Introduction

The fuzzy concept has invaded almost all branches of Mathematics since the introduction of the concept of fuzzy sets by Zadeh [32]. Fuzzy sets have applications in many fields such as information [24] and control [25]. The theory of fuzzy topological spaces was introduced and developed by Chang [7] and since then various notions in classical topology have been extended to fuzzy topological spaces. The concept of fuzzy almost continuous mapping [1], fuzzy completely continuous mapping [4], fuzzy  $\delta$ -continuous [15] and fuzzy  $R$  mapping were introduced by Azad [1], Bhaumik [4] and Mukerjee [20] and Ganguly and Saha [15, 16] respectively. The initiations of  $e$ -open sets,  $e^*$ -open sets,  $a$ -open sets,  $e$ -continuity and  $e$ -compactness in topological spaces are due to Ekici [10–14]. In 2014, the concept

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of fuzzy  $e$ -open sets and fuzzy  $e$ -continuity and separation axioms and other properties were defined by Seenivasan and Kamala [27].

In this paper, we introduce a new class of mappings viz. fuzzy almost generalized  $e$ -continuous mapping and establish some of their properties. We also introduce fuzzy  $fT_{\frac{1}{2}}e$ -space, fuzzy generalized  $e$ -compact space, fuzzy  $ge$  space, fuzzy regular  $ge$  space. It is seen that a fuzzy almost generalized  $e$ -continuous surjection image of a fuzzy generalized  $e$ -compact space is fuzzy nearly compact.

## 2. Preliminaries

A family of fuzzy sets of  $X$  is called a fuzzy topology on  $X$  if  $0$  and  $1$  belong to  $\tau$  and  $\tau$  is closed with respect to arbitrary union and finite intersection. The members of  $\tau$  are called fuzzy open sets and their complements are fuzzy closed sets. Throughout this paper,  $(X, \tau)$ ,  $(Y, \sigma)$ , and  $(Z, \eta)$ , (or simply  $X$ ,  $Y$  and  $Z$ ) mean fuzzy topological space (abbreviated as fts) on which no separation axioms are assumed unless otherwise mentioned. We denote the closure and interior of a fuzzy set  $\lambda$  in  $X$  by  $Cl(\lambda)$  and  $Int(\lambda)$ , whereas the constant fuzzy sets taking on the values  $0$  and  $1$  on  $X$  are denoted by  $0_X$  and  $1_X$  respectively.

This section contains some basic definitions and preliminary results which will be needed in the sequel.

**Definition 2.1** [21] A fuzzy subset  $\lambda$  in an fts  $X$  is called

- (i) quasi-coincident ( $q$ -coincident, for short) with a fuzzy point  $x_p$  (where  $x$  is the support and  $p$ ,  $0 < p \leq 1$ , is the value of the point) iff  $p + \lambda(x) > 1$ .
- (ii)  $q$ -coincident with a fuzzy set  $\mu$ , denoted by  $\lambda q \mu$ , iff  $\exists x \in X$  such that  $\lambda(x) + \mu(x) > 1$ .
- (iii)  $q$ -neighborhood ( $q$ - nbd, for short) of fuzzy point  $x_p$  iff  $\exists$  a fuzzy open set  $\mu$  such that  $x_p q \mu \leq \lambda$ .

**Definition 2.2** A fuzzy subset  $\lambda$  in an fts  $(X, \tau)$  is called

- (i) fuzzy semi-open set [1] if  $\lambda \leq ClInt(\lambda)$  and a fuzzy semi-closed set if  $IntCl(\lambda) \leq \lambda$ .
- (ii) fuzzy pre-open set [3] if  $\lambda \leq IntCl(\lambda)$  and a pre-closed set if  $ClInt(\lambda) \leq \lambda$ .
- (iii) fuzzy regular open ( $fro$ , for short) [1] if  $\lambda = IntCl(\lambda)$  and a regular closed set if  $\lambda = ClInt(\lambda)$ .

**Definition 2.3** [28] The fuzzy  $\delta$  interior of subset  $\lambda$  of  $X$  is the union of all fuzzy regular open sets contained in  $\lambda$ .

**Definition 2.4** [31] A subset  $\lambda$  is called fuzzy  $\delta$  open if  $\lambda = \delta Int(\lambda)$ . The compliment of fuzzy  $\delta$  open set is called fuzzy  $\delta$  closed (i.e.,  $\lambda = \delta Cl(\lambda)$ .)

**Definition 2.5** [27] Let  $\lambda$  be a fuzzy set of a fuzzy topological space  $X$ . Then  $\lambda$  is called

- (i) a fuzzy  $e$ -open set of  $X$  if  $\lambda \leq Cl(\delta Int(\lambda)) \vee Int(\delta Cl(\lambda))$ .
- (ii) a fuzzy  $e$ -closed set of  $X$  if  $Cl(\delta Int(\lambda)) \vee Int(\delta Cl(\lambda)) \leq \lambda$ .

**Lemma 2.6** [27] In a fuzzy topological space  $X$ ,

- 1 Any union of fuzzy  $e$ -open sets is a fuzzy  $e$ -open set.
- 2 Any intersection of  $e$ -closed sets is a fuzzy  $e$ -closed set.

**Definition 2.7** [27] Let  $(X, \tau)$  be a fuzzy topological space. Let  $\lambda$  be a fuzzy set of a fuzzy topological space  $X$ .

- (i)  $eInt(\lambda) = \bigvee \{ \mu \in I^X : \mu \leq \lambda, \mu \text{ is a } feo \text{ set} \}$  is called the fuzzy  $e$ -interior of  $\lambda$ .
- (ii)  $eCl(\lambda) = \bigwedge \{ \mu \in I^X : \mu \geq \lambda, \mu \text{ is a } fe\text{-closed set} \}$  is called the fuzzy  $e$ -closure of  $\lambda$ .

**Definition 2.8** Let  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$  be a mapping from a fts  $(X, \tau_1)$  to another  $(Y, \tau_2)$ . Then  $f$  is called

- (1) fuzzy continuous [7] if  $f^{-1}(\lambda)$  is fuzzy open set in  $X$  for any fuzzy open set  $\lambda$  in  $Y$ .
- (2) fuzzy almost continuous [1] if  $f^{-1}(\lambda)$  is fuzzy open set in  $X$  for any fuzzy regular open (fro, for short) set  $\lambda$  in  $Y$ .
- (3) fuzzy  $e$ -continuous [27] if  $f^{-1}(\lambda)$  is a  $feo$  set in  $X$  for any fuzzy open set  $\lambda$  in  $Y$ .
- (4) fuzzy  $e$ -irresolute [27] if  $f^{-1}(\lambda)$  is a  $feo$  set in  $X$  for any  $feo$  set  $\lambda$  in  $Y$ .

**Definition 2.9** A fts  $(X, \tau)$  is said to be

- (1) a fuzzy nearly compact [16] iff every fuzzy regular open cover of  $X$  has a finite subcover.
- (2) a fuzzy Hausdorff [17] if for every pair of fuzzy points  $x_p, y_q$  in  $X$  with distinct supports,  $\exists$  open fuzzy sets  $U$  and  $V$  such that  $x_p \in U, y_q \in V$  and  $U \wedge V = 0_X$ .

**Definition 2.10** [7] A family  $\Lambda$  of fuzzy sets in a fuzzy space  $X$  is said to be a cover of fuzzy set  $\mu$  of  $X$  if and only if  $\mu \leq \bigvee \{ \lambda_i : \lambda_i \in \Lambda \}$ .  $\Lambda$  is called fuzzy open cover if each member  $\lambda_i$  is a fuzzy open set. A sub cover of  $\Lambda$  is a subfamily of  $\Lambda$  which is also a cover of  $\mu$ .

**Definition 2.11** [7] Let  $(X, \tau)$  be a fuzzy topological space and let  $\mu \in I^X$ .  $\mu$  is said to be a fuzzy compact set if for every fuzzy open cover of  $\mu$  has a finite subcover of  $\mu$ .

**Definition 2.12** [29] A family  $\lambda$  of fuzzy sets in a fuzzy space  $X$  is said to be an  $e$ -open cover of a fuzzy set  $\mu$  of  $X$  if  $\mu \leq \bigvee \{ B : B \in \lambda \}$ . Each member of  $\lambda$  is an  $e$ -open cover of a fuzzy set. A sub cover of  $\lambda$  is a subfamily which is also a cover.

**Definition 2.13** [29] A fuzzy topological space  $(X, \tau)$  is fuzzy  $e$ -compact space if every fuzzy  $e$ -open cover of  $X$  has a finite subcover.

### 3. Fuzzy almost generalized $e$ -continuous mappings

Now, we introduce the following definitions.

**Definition 3.1** A fuzzy subset  $\lambda$  in a fts  $X$  is called

- (i) fuzzy generalized  $e$ -closed (in short, fgec) set iff  $fCl(\lambda) \leq \mu$ , whenever  $\lambda \leq \mu$  and  $\mu$  is feo set.
- (ii) fuzzy generalized  $e$ -open (in short, fgeo) set iff  $\mu \leq fInt(\lambda)$  whenever  $\mu \leq \lambda$  and  $\mu$  is fec set.

The complement of a fuzzy generalized  $e$ -closed set is called a fuzzy generalized  $e$ -open set.

**Definition 3.2** Let  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$  be a mapping from a fts  $(X, \tau_1)$  to another  $(Y, \tau_2)$ . Then  $f$  is called

- (1) fuzzy generalized  $e$  (in short, fge)-continuous mapping iff  $f^{-1}(\lambda)$  is fgeo set in  $X$  for every fuzzy open set  $\lambda$  in  $Y$ .
- (2) fuzzy generalized  $e$  (in short, fge)-irresolute mapping iff  $f^{-1}(\lambda)$  is fgeo in  $X$  for every fgeo set  $\lambda$  in  $Y$ .

**Definition 3.3** An fts  $(X, \tau)$  is said to be  $fT_{\frac{1}{2}}e$ -space if every fgec set in  $X$  is fuzzy

closed.

**Definition 3.4** A mapping  $f$  from an fts  $(X, \tau_1)$  to another fts  $(Y, \tau_2)$  is said to be fuzzy almost generalized  $e$  (in short, fage)-continuous mapping if  $f^{-1}(\lambda)$  is fgeo in  $X$  for every fuzzy regular open set  $\lambda$  in  $Y$ .

Equivalently,  $f$  is said to be fuzzy almost generalized  $e$ -continuous mapping iff  $f^{-1}(\lambda)$  is fgec in  $X$  for every fuzzy regular closed set  $\lambda$  in  $Y$ .

**Example 3.5** Let  $\lambda, \mu, \gamma$  and  $\delta$  be fuzzy subsets of  $X = Y = \{a, b, c\}$  are defined as follows:

$$\begin{aligned}\lambda(a) &= 0.4, \lambda(b) = 0.6, \lambda(c) = 0.5; \\ \mu(a) &= 0.6, \mu(b) = 0.4, \mu(c) = 0.4; \\ \gamma(a) &= 0.6, \gamma(b) = 0.6, \gamma(c) = 0.5; \\ \delta(a) &= 0.4, \delta(b) = 0.4, \delta(c) = 0.4.\end{aligned}$$

Then  $\tau_1 = \{0, 1, \lambda, \mu, \gamma, \delta\}$  and  $\tau_2 = \{0, 1, \delta\}$  are fuzzy topologies on  $X$ . Consider the mapping  $f : (X, \tau_1) \rightarrow (X, \tau_2)$  defined by  $f(x) = x, \forall x \in X$ . Then for every fuzzy regular open set  $\delta$  in  $X$ ,  $f^{-1}(\delta)$  is fuzzy generalized  $e$ -open (fgeo) set in  $X$ . Thus  $f$  is a fuzzy almost generalized  $e$ -continuous mapping from an fts  $(X, \tau_1)$  to  $(X, \tau_2)$ .

**Remark 1** It follows from Definition 3.2 and 3.4, that every fge continuous mapping is an fage continuous mapping, but the converse may not be true is shown by the following example.

**Example 3.6** Let  $\lambda, \mu, \gamma$  and  $\delta$  be fuzzy subsets of  $X = \{a, b, c\}$  are defined as follows:

$$\begin{aligned}\lambda(a) &= 0.3, \lambda(b) = 0.4, \lambda(c) = 0.5; \\ \mu(a) &= 0.6, \mu(b) = 0.5, \mu(c) = 0.5; \\ \omega(a) &= 0.4, \omega(b) = 0.4, \omega(c) = 0.4; \\ \delta(a) &= 0.5, \delta(b) = 0.4, \delta(c) = 0.5.\end{aligned}$$

Now,  $\tau_1 = \{0, 1, \lambda, \mu\}$  and  $\tau_2 = \{0, 1, \omega, \delta\}$  are fuzzy topologies on  $X$ .

Consider the mapping  $f : (X, \tau_1) \rightarrow (X, \tau_2)$  defined by  $f(x) = x, \forall x \in X$ . Then for every fro set  $\delta$  in  $Y$ ,  $f^{-1}(\delta)$  is fgeo set in  $X$ . Hence,  $f$  is fage-continuous mapping. Since  $\omega$  is fuzzy open set in  $Y$ , but  $f^{-1}(\omega)$  is not fgeo in  $X$ . Hence  $f$  is not fge-continuous. Thus fage-continuous mapping  $\not\Rightarrow$  fge-continuous mapping.

**Definition 3.7** A fuzzy point  $x_\alpha$  in a fuzzy topological space  $X$  is called a fuzzy  $e$ -cluster point of a fuzzy set  $A$  in  $X$  if every fuzzy  $e$ - $q$ -nbd of  $x_\alpha$  is  $q$ -coincident with  $A$ .

The union of all fuzzy  $e$ -cluster points of  $A$  is called the fuzzy  $e$ -closure of  $A$  and is denoted by  $eCl(A)$ .

**Theorem 3.8** Let  $f : X \rightarrow Y$  be a fuzzy mapping. Then the following statements are equivalent:

- (1)  $f$  is fage continuous.
- (2) for each fuzzy point  $x_p$  in  $X$  and each fro  $q$ -nbd  $\mu$  of  $f(x_p)$ , there is a  $f(x_p)$   $q$ -nbd  $\lambda$  of  $x_p$  such that  $f(\lambda) \leq \mu$ .
- (3) for each fuzzy set  $\lambda$  in  $X$ ,  $f(geCl(\lambda)) \leq eClf(\lambda)$ .
- (4) for each fuzzy set  $\mu$  in  $Y$ ,  $geCl(f^{-1}(\mu)) \leq f^{-1}(eCl(\mu))$ .

**Proof.** (1)  $\Rightarrow$  (2): Let  $x_p$  be a fuzzy point of  $x$ . Then  $f(x_p)$  is a fuzzy point in  $Y$ . Now, let  $\mu \in Y$  be a fuzzy regular open set such that  $f(x_p)q\mu$ . For,  $\lambda = f^{-1}(\mu)$  as  $f$  is fage continuous, we have  $\lambda$  is fgeo set of  $X$  and  $x_p \in \lambda$ . Therefore,  $f(\lambda) = f(f^{-1}(\mu)) \leq \mu$ .

(2)  $\Rightarrow$  (3): Let  $x_p \in geCl(\lambda)$ . Then  $x_pq\lambda$  and  $f(x_p)qf(\lambda)$  implies  $f(x_p) \in eCl(f(\lambda))$  and  $x_p \in f^{-1}(eCl(f(\lambda)))$ . Therefore,  $geCl(\lambda) \leq f^{-1}(eClf(\lambda))$ .

(3)  $\Rightarrow$  (4) and (4)  $\Rightarrow$  (1) are clear. ■

**Theorem 3.9** A mapping from an  $fT_{\frac{1}{2}}e$ -space to another fts is fuzzy almost continuous if it is fage-continuous.

**Proof.** Let  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$  be fage-continuous mapping. Let  $\lambda$  be any fuzzy regular closed set in  $Y$ . Since  $f$  is fage-continuous,  $f^{-1}(\lambda)$  is fgec in  $X$ . As  $X$  is  $fT_{\frac{1}{2}}e$ -space,  $f^{-1}(\lambda)$  is closed in  $X$ . Thus  $f$  is fuzzy almost continuous. ■

**Definition 3.10** A mapping  $f : X \rightarrow Y$  is called fge (resp. fe)-irresolute mapping if  $f^{-1}(\lambda)$  is fgeo (resp. feo) subset of  $X$  for every fgeo (resp. feo) set  $\lambda$  in  $Y$ .

**Example 3.11** Let  $\lambda, \mu, \gamma, \delta,$  and  $\omega$  be fuzzy subsets of  $X = \{a, b, c\}$  and defined as follows:

$$\begin{aligned} \lambda(a) &= 0.3, \lambda(b) = 0.4, \lambda(c) = 0.5; \\ \mu(a) &= 0.6, \mu(b) = 0.5, \mu(c) = 0.5; \\ \gamma(a) &= 0.6, \gamma(b) = 0.5, \gamma(c) = 0.4; \\ \delta(a) &= 0.3, \delta(b) = 0.4, \delta(c) = 0.4; \\ \omega(a) &= 0.8, \omega(b) = 0.4, \omega(c) = 0.5. \end{aligned}$$

Then  $\tau_1 = \{0, 1, \lambda, \mu, \gamma, \delta\}$  and  $\tau_2 = \{0, 1, \omega\}$  are fuzzy topologies on  $X$ . Consider the mapping  $f : (X, \tau_1) \rightarrow (X, \tau_2)$  defined by  $f(x) = x, \forall x \in X$ . Then for every fuzzy generalized  $e$ -open (fgeo)(resp. feo) set  $\omega$  in  $X, f^{-1}(\omega)$  is fuzzy generalized  $e$ -open (fgeo)(resp. feo) set in  $X$ . Thus,  $f$  is a fuzzy generalized  $e$  (fge)(resp. fe)-irresolute mapping from an fts  $(X, \tau_1)$  to  $(X, \tau_2)$ .

**Theorem 3.12** Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two fuzzy mappings. Then  $g \circ f : X \rightarrow Z$  is fage-continuous if  $f$  is fge-irresolute and  $g$  is fage-continuous.

**Proof.** Let  $\lambda$  be any fuzzy regular open set in  $Z$ . Since  $g$  is fage-continuous,  $g^{-1}(\lambda)$  is fgeo in  $Y$ . As  $f$  is fge-irresolute,  $f^{-1}(g^{-1}(\lambda))$  is fgeo in  $X$ . This implies  $(g \circ f)^{-1}(\lambda)$  is fgeo in  $X$ . Thus,  $g \circ f$  is fage-continuous. ■

**Theorem 3.13** If  $f : X \rightarrow Y$  is an fuzzy continuous mapping and  $g : Y \rightarrow Z$  is an fage-continuous mapping, where  $Y$  is  $fT_{\frac{1}{2}}e$ -space, then  $g \circ f : X \rightarrow Z$  is fuzzy almost continuous mapping.

**Proof.** Let  $\lambda$  be any fuzzy regular closed set in  $Z$ . Since  $g$  is fage-continuous mapping,  $g^{-1}(\lambda)$  is fgec in  $Y$ . As  $Y$  is  $fT_{\frac{1}{2}}e$ -space,  $g^{-1}(\lambda)$  is fuzzy closed in  $Y$ . This implies  $Y - g^{-1}(\lambda)$  is fuzzy open in  $Y$ . As  $f$  is fuzzy continuous,  $f^{-1}(Y - g^{-1}(\lambda))$  is fuzzy open in  $X \Rightarrow X - f^{-1}(g^{-1}(\lambda))$  is fuzzy open in  $X \Rightarrow f^{-1}(g^{-1}(\lambda))$  is fuzzy closed in  $X \Rightarrow (g \circ f)^{-1}(\lambda)$  is fuzzy closed in  $X$ . Hence,  $g \circ f$  is fuzzy almost continuous. ■

#### 4. Fuzzy generalized $e$ -compact space and fuzzy regular $ge$ -space

In this section, we introduce the definition of fuzzy generalized  $e$ -compact (fge compact space) and fuzzy regular  $ge$ -space. Also some interesting theorems involving fge compact space and fuzzy regular  $ge$ -space are established here.

**Definition 4.1** An fts  $(X, \tau)$  is said to be fge-compact space if every fgeo cover of  $X$  has a finite subcover.

**Theorem 4.2** An fage continuous surjection image of a fge compact space is fuzzy nearly compact.

**Proof.** Let  $f : X \rightarrow Y$  be an fage continuous surjection mapping and  $\{\lambda_i\}_{i \in \Lambda}$  be any fro cover of  $Y$ . Then  $\{f^{-1}(\lambda_i)\}_{i \in \Lambda}$  is a family of fgeo cover of  $X$ .  $X$  is fge compact. So, there exists a finite subset  $\Lambda_0$  of  $\Lambda$  such that  $\{f^{-1}(\lambda_i)\}_{i \in \Lambda_0}$  covers  $X$ . Since  $f$  is 1-1, onto,  $\{\lambda_i\}_{i \in \Lambda_0}$  will be finite fro subcover of the fro cover  $\{\lambda_i\}_{i \in \Lambda}$  of  $Y$ . So,  $Y$  is fuzzy nearly compact. ■

**Definition 4.3** An fts  $(X, \tau)$  is said to be fuzzy regular  $ge$ -space if every fgeo set in  $X$  is fro.

**Remark 2** There are spaces which are not fuzzy regular  $ge$ -spaces.

**Example 4.4** Let  $\lambda, \mu, \gamma$  and  $\delta$  be fuzzy subsets of  $X = \{a, b, c\}$  defined as follows:

$$\begin{aligned}\lambda(a) &= 0.3, \lambda(b) = 0.4, \lambda(c) = 0.5; \\ \mu(a) &= 0.7, \mu(b) = 0.4, \mu(c) = 0.5; \\ \gamma(a) &= 0.8, \gamma(b) = 0.5, \gamma(c) = 0.5; \\ \delta(a) &= 0.8, \delta(b) = 0.4, \delta(c) = 0.5.\end{aligned}$$

Then  $\tau = \{0, 1, \lambda, \mu\}$  is a fuzzy topology on  $X$ . Since  $\gamma$  and  $\delta$  are fgeo sets in  $X$ , but not fuzzy regular open in  $(X, \tau)$ . Hence the topological space  $(X, \tau)$  is not fuzzy regular  $ge$ -space.

**Definition 4.5** An fts  $(X, \tau)$  is said to be fuzzy  $ge$ -space if every fgeo set in  $X$  is fuzzy open.

**Remark 3** There are spaces which are not fuzzy  $ge$ -spaces is shown by the example below.

**Example 4.6** Let  $\lambda, \mu, \gamma$  and  $\delta$  be fuzzy subsets of  $X = \{a, b, c\}$  defined as follows:

$$\begin{aligned}\lambda(a) &= 0.3, \lambda(b) = 0.5, \lambda(c) = 0.2; \\ \mu(a) &= 0.4, \mu(b) = 0.5, \mu(c) = 0.5; \\ \gamma(a) &= 0.5, \gamma(b) = 0.7, \gamma(c) = 0.5; \\ \delta(a) &= 0.4, \delta(b) = 0.6, \delta(c) = 0.5.\end{aligned}$$

Then  $\tau = \{0, 1, \lambda, \mu\}$  is a fuzzy topology on  $X$ . Since  $\gamma$  and  $\delta$  are fgeo sets in  $X$  but not fuzzy open sets in  $X$ . This implies the fuzzy topological space  $(X, \tau)$  is not fuzzy  $ge$ -space.

**Remark 4** It is obvious from Definition 4.3 and 4.5 that every fuzzy regular  $ge$  space is fuzzy  $ge$  space but not the converse is shown by the following example.

**Example 4.7** Let  $\lambda$  and  $\mu$  be fuzzy subsets of  $X = \{a, b, c\}$  defined as follows:

$$\begin{aligned}\lambda(a) &= 0.3, \lambda(b) = 0.5, \lambda(c) = 0.2; \\ \mu(a) &= 0.5, \mu(b) = 0.5, \mu(c) = 0.5.\end{aligned}$$

Then  $\tau = \{0, 1, \lambda, \mu\}$  is a fuzzy topology on  $X$ . Now  $(X, \tau)$  is fuzzy  $ge$ -space but not fuzzy regular  $ge$ -space since,  $\lambda$  is fuzzy open and fgeo but not fro set in  $(X, \tau)$ .

**Theorem 4.8** Let  $f : X \rightarrow Y$  be fge continuous surjection mapping. Then  $Y$  is fuzzy connected if  $X$  is fuzzy connected space and fuzzy regular  $ge$ -space.

**Proof.** Let  $X$  be a fuzzy connected space and  $Y$  be not fuzzy connected. Then there exists a fuzzy subset  $\lambda$  of  $Y$  such that  $\lambda \neq 0_Y, 1_Y$  and  $\lambda$  is both fuzzy open and fuzzy closed in  $Y \Rightarrow f^{-1}(\lambda)$  is both fgeo and fgec in  $X \Rightarrow f^{-1}(\lambda)$  is both  $fro$  and  $frc$  in  $X \Rightarrow f^{-1}(\lambda)$  is both fuzzy open and fuzzy closed in  $X$ . Also,  $f^{-1}(\lambda) \neq 0_X, 1_X$ . This implies that  $X$  is not a fuzzy connected space, a contradiction. Therefore,  $Y$  is fuzzy connected. ■

**Theorem 4.9** Let  $f : X \rightarrow Y$  be an fage continuous surjective mapping and  $X$  be fuzzy regular  $ge$ -space as well as fuzzy nearly compact space. Then  $Y$  is fuzzy nearly compact.

**Proof.** Let  $\{\lambda_i\}_{i \in \Lambda}$  be any fro cover of  $Y$ . Since  $f$  is fage continuous,  $f^{-1}(\lambda_i)_{i \in \Lambda}$  is a family of fgeo sets in  $X$ .  $X$  is fuzzy regular  $ge$ -space. So,  $f^{-1}(\lambda_i)_{i \in \Lambda}$  is a family of  $fro$  sets in  $X$ .  $X$  is fuzzy nearly compact. So, there exists a finite subset  $\Lambda_0$  of  $\Lambda$  such that  $X \leq [f^{-1}(\lambda_i)_{i \in \Lambda_0}]$ . Then  $Y = f(X) \leq (\lambda_i)_{i \in \Lambda_0}$ . So,  $\{\lambda_i\}_{i \in \Lambda_0}$  is a finite subcover of  $fro$  sets of  $Y$ . So,  $Y$  is fuzzy nearly compact. ■

**Theorem 4.10** If  $f : X \rightarrow Y$  is an fge continuous injective mapping and  $Y$  is a fuzzy Hausdorff space, then  $X$  is fuzzy Hausdorff if it is fuzzy regular  $ge$ -space.

**Proof.** Let  $x_p, y_q$  be two distinct fuzzy points in  $X$ . Then  $f(x_p) \neq f(y_q)$  in  $Y$ . Since  $Y$  is fuzzy Hausdorff, there exists fuzzy open nbds  $U$  and  $V$  of  $f(x_p)$  and  $f(y_q)$  respectively such that  $U \wedge V = 0_X$ . Since  $f$  is an fge continuous,  $f^{-1}(U)$  and  $f^{-1}(V)$  are fgeo in  $X \Rightarrow f^{-1}(U)$  and  $f^{-1}(V)$  are  $fro$  in  $X$ . [Since  $X$  is fuzzy regular  $ge$ -space]  $\Rightarrow f^{-1}(U)$  and  $f^{-1}(V)$  are fuzzy open in  $X$  and contains respectively the fuzzy points  $x_p$  and  $y_q$ . Now,  $x_p \in f^{-1}(U) = \lambda$ , say  $y_q \in f^{-1}(V) = \mu$ , say. So,  $\lambda \wedge \mu = f^{-1}(U) \wedge f^{-1}(V) = f^{-1}(U \wedge V) = f^{-1}(0_X) = 0_X$ . Thus,  $X$  is fuzzy Hausdorff. ■

## 5. Conclusion

It is interesting to work under a new class of mappings viz. fuzzy almost generalized  $e$ -continuous mapping, fuzzy  $fT_{\frac{1}{2}}$ - $e$ -space, fuzzy generalized  $e$ -compact space, fuzzy  $ge$ -space, fuzzy regular  $ge$ -space. It is seen that a fuzzy almost generalized  $e$ -continuous surjection image of a fuzzy generalized  $e$ -compact space is fuzzy nearly compact.

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