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γ_μ-Lindelöf generalized topological spaces

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Abstract. In this paper we have introduced new types of sets termed as ω_{γ_μ}-open sets with the help of an operation and a generalized topology. We have also defined a notion of γ_μ-Lindelöf spaces and discussed some of its basic properties.

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1. Introduction

In 1979, Kasahara [10] introduced the notion of an operation on a topological space and the concept of an α-closed graph of a function. After that, Janković defined [9] the concept of α-closed sets and investigated some properties of functions with α-closed graphs. On the other hand, in 1991, Ogata [13] introduced the notion of γ-open sets to investigate some new separation axioms of a topological space. In 1982, Hdeib [7] introduced the notion of ω-open sets as a weak form of open sets in topological spaces. By using ω-open sets, he obtained some improvements of characterizations and preservation theorems of Lindelöf spaces. Analogously, by using preopen sets, Hdeib and Sarsak [8] defined ω-preopen sets and obtained further properties of strongly Lindelöf spaces due to Mashhour et al. [11]. As generalizations of Lindelöf spaces, Ganster [6] introduced and investigated α-Lindelöf spaces and semi-Lindelöf spaces, while the notion of nearly Lindelöf space was studied in [1, 2, 5, 12].

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In this paper our aim is to study an operation based on ω-open like sets, where the operation is defined on a collection of generalized open sets. The most common properties of different open like sets or weakly open sets are that they are closed under arbitrary unions and contain the null set. Observing these, Császár introduced the concept of generalized open sets. We now recall some notions defined in [3]. Let X be a non-empty set. A subcollection $\mu \subseteqq \mathcal{P}(X)$ (where $\mathcal{P}(X)$ denotes the power set of X) is called a generalized topology [3] (briefly, GT) if $\varnothing \in \mu$ and any union of elements of μ belongs to μ. A set X with a GT μ on the set X is called a generalized topological space (briefly, GTS) and is denoted by (X, μ). For a GTS (X, μ), if $X \in \mu$ then (X, μ) is known as a strong GTS. The elements of μ are called μ-open sets and μ-closed sets are their complements. The μ-closure of a set $A \subseteqq X$ is denoted by $c_\mu(A)$ and defined by the smallest μ-closed set containing A which is equivalent to the intersection of all μ-closed sets containing A. We use the symbol $i_\mu(A)$ to mean the μ-interior of A and it is defined as the union of all μ-open sets contained in A i.e., the largest μ-open set contained in A (see [3, 4]).

2. ω_{γ_μ}-open sets

Definition 2.1 [14] Let (X, μ) be a GTS. An operation γ_μ on a generalized topology μ is a mapping from μ to $\mathcal{P}(X)$ with $G \subseteqq \gamma_\mu(G)$ for each $G \in \mu$. This operation is denoted by $\gamma_\mu : \mu \to \mathcal{P}(X)$.

Note that $\gamma_\mu(G)$ and G^{γ_μ} are two different notations for the same set.

Definition 2.2 [14] Let (X, μ) be a GTS and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation on μ. A subset G of a GTS (X, μ) is called γ_μ-open if, for each point x of G, there exists a μ-open set U containing x such that $\gamma_\mu(U) \subseteqq G$.

A subset of a GTS (X, μ) is called γ_μ-closed if its complement is γ_μ-open in (X, μ). We shall use the symbol γ_μ to mean the collection of all γ_μ-open sets of a GTS (X, μ).

Definition 2.3 Let (X, μ) be a GTS and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation on μ. A subset A of a GTS (X, μ) is said to be ω_{γ_μ}-open if for each $x \in A$, there exists a γ_μ-open set U containing x such that $U \setminus A$ is at most countable. The complement of an ω_{γ_μ}-open set is called an ω_{γ_μ}-closed set.

Every γ_μ-open set is an ω_{γ_μ}-open set but the converse is not necessarily true follows from the next example. Furthermore if $\gamma_\mu = id_\mu$, then every ω_{γ_μ}-open set is an ω_μ-open set.

***Example* 2.4** Let $\mathbb{R}$ be the set of real line and $\mu = \{A \subseteqq \mathbb{R} : 0 \in A\} \cup \{\varnothing\}$. Then $(\mathbb{R}, \mu)$ is a GTS. Now $\gamma_\mu : \mu \to \mathcal{P}(X)$ defined by

$$\gamma_\mu(A) = \begin{cases} A, & \text{if } 0 \in A \\ \varnothing, & \text{otherwise} \end{cases}$$

is an operation. It is easy to check that $\{1\}$ is an ω_{γ_μ}-open set but not a γ_μ-open set.

Theorem 2.5 Let (X, μ) be a GTS and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation on μ. A subset A of a generalized topological space (X, μ) is ω_{γ_μ}-open if and only if for each $x \in A$, there exist a γ_μ-open set U containing x and a countable set C of X such that $U \setminus C \subseteqq A$.

Proof. Let A be ω_{γ_μ}-open and $x \in A$. Then there exists a γ_μ-open set U containing x such that $U \setminus A$ is countable. Let $C = U \setminus A$. Then we have $U \setminus C \subseteqq A$.

Conversely, let $x \in A$. Then there exist an $U \in \gamma_\mu$ containing x and a countable set C such that $U \setminus C \subseteqq A$. Therefore, $U \setminus A \subseteqq C$ and $U \setminus A$ is countable. ■

Theorem 2.6 Let (X, μ) be a GTS and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation on μ and $C \subseteqq X$. If C is ω_{γ_μ}-closed, then $C \subseteqq K \cup B$ for some γ_μ-closed set K and a countable set B.

Proof. If C is ω_{γ_μ}-closed, then $X \setminus C$ is ω_{γ_μ}-open. Hence for each $x \in X \setminus C$, there exist a γ_μ-open set U containing x and a countable set B such that $U \setminus B \subseteqq X \setminus C$. Thus $C \subseteqq X \setminus (U \setminus B) = X \setminus (U \cap (X \setminus B)) = (X \setminus U) \cup B$. Let $K = X \setminus U$. Then K is a γ_μ-closed set such that $C \subseteqq K \cup B$. ■

Definition 2.7 [15] Let (X, μ) be a GTS. An operation $\gamma_\mu : \mu \to \mathcal{P}(X)$ is said to be regular if for each point $x \in X$ and any two μ-open sets U and V of X containing x, there exists a μ-open set W containing x such that $\gamma_\mu(W) \subseteqq \gamma_\mu(U) \cap \gamma_\mu(V)$.

Theorem 2.8 Let (X, μ) be a GTS and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation on μ. Then the family of all ω_{γ_μ}-open sets forms a GT on X. If further γ_μ is regular, then
(a) the family of all ω_{γ_μ}-open sets is closed under finite intersections.
(b) $(\omega_{\gamma_\mu})_{\omega_{\gamma_\mu}} = \omega_{\gamma_\mu}$.

Proof. It is obvious that $\varnothing$ is an ω_{γ_μ}-open set. Let $\{A_\alpha : \alpha \in \Lambda\}$ be a family of ω_{γ_μ}-open sets and $x \in \cup\{A_\alpha : \alpha \in \Lambda\}$. Then $x \in A_\alpha$ for some $\alpha \in \Lambda$. Thus, there exists a γ_μ-open set U such that $U \setminus A_\alpha$ is a countable set. Hence, $U \setminus \cup\{A_\alpha : \alpha \in \Lambda\}$ is a countable set.

(a) Let A and B be two ω_{γ_μ}-open sets and $x \in A \cap B$. Then there exists γ_μ-open sets U and V containing x such that $U \setminus A$ and $V \setminus B$ are both countable. Clearly $U \cap V$ is a γ_μ-open set (as γ_μ is a regular operation) containing x. Also,

$$\begin{aligned}(U \cap V) \setminus (A \cap B) &= (U \cap V) \cap [(X \setminus A) \cup (X \setminus B)] \\ &\subseteqq [U \cap V \cap (X \setminus A)] \cup [U \cap V \cap (X \setminus B)] \\ &\subseteqq [U \cap (X \setminus A)] \cup [V \cap (X \setminus B)] \\ &= (U \setminus A) \cup (V \setminus B)\end{aligned}$$

. Since $U \setminus A$ and $V \setminus B$ are countable, $(U \setminus A) \cup (V \setminus B)$ is countable.

(b) Let $A \in (\omega_{\gamma_\mu})_{\omega_{\gamma_\mu}}$. Then, by Theorem 2.5, there exist an ω_{γ_μ}-open set U containing x for each $x \in A$ and a countable set C such that $U \setminus C \subseteqq A$. Furthermore, there exist a γ_μ-open set V and a countable set D such that $V \setminus D \subseteqq U$. Therefore, $V \setminus (C \cup D) \subseteqq [V \setminus D] \setminus C \subseteqq U \setminus C \subseteqq A$. Since $C \cup D$ is countable, we have A an ω_{γ_μ}-open set. ■

3. γ_μ-Lindelöf spaces

Definition 3.1 Let (X, μ) be a strong GTS and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation on μ. A subset A of (X, μ) is said be γ_μ-Lindelöf (resp. ω_{γ_μ}-Lindelöf) relative to X if for every cover $\{U_\alpha : \alpha \in \Lambda\}$ of A by γ_μ-open (resp. ω_{γ_μ}-open) subsets of X, there exists a countable subset Λ_0 of Λ such that $A \subseteqq \cup\{U_\alpha : \alpha \in \Lambda_0\}$. If $A = X$, then the γ_μ-Lindelöf (resp. ω_{γ_μ}-Lindelöf) subset A is called a γ_μ-Lindelöf (resp. ω_{γ_μ}-Lindelöf) space.

Remark 1 *If $\gamma_\mu = id_\mu$, then the notion of a γ_μ-Lindelöf space reduces to a μ-Lindelöf space* [16].

Theorem 3.2 Let (X,μ) be a strong GTS and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation on μ. A subset K of a strong GTS (X,μ) is γ_μ-Lindelöf if and only if K is ω_{γ_μ}-Lindelöf.

Proof. Suppose that a subset K of a strong GTS (X,μ) is γ_μ-Lindelöf relative to X. Let $\{U_\alpha : \alpha \in \Lambda\}$ be a cover of K by ω_{γ_μ}-open sets of (X,μ). For each $x \in K$, there exists $\alpha(x) \in \Lambda$ such that $x \in U_{\alpha(x)}$. Since $U_{\alpha(x)} \in \omega_{\gamma_\mu}$, there exists $V_{\alpha(x)} \in \gamma_\mu$ containing x such that $V_{\alpha(x)} \setminus U_{\alpha(x)}$ is a countable set. Then the family $\{V_{\alpha(x)} : x \in K\}$ is a cover of K by γ_μ-open sets in (X,μ). Since K is γ_μ-Lindelöf, there exists a countable subset, say, $\alpha(x_1), \alpha(x_2), ..., \alpha(x_n), ...$ such that $K \subseteqq \cup\{V_{\alpha(x_i)} : i \in \mathbb{N}\}$. Now, we have

$$K \subseteqq \cup_{i\in\mathbb{N}}\{(V_{\alpha(x_i)} \setminus U_{\alpha(x_i)}) \cup U_{\alpha(x_i)}\} = [\cup_{i\in\mathbb{N}}(V_{\alpha(x_i)} \setminus U_{\alpha(x_i)})] \cup [\cup_{i\in\mathbb{N}} U_{\alpha(x_i)}]$$

and hence $K \subseteqq [\cup_{i\in\mathbb{N}}(V_{\alpha(x_i)} \setminus U_{\alpha(x_i)}) \cap K] \cup [\cup_{i\in\mathbb{N}} U_{\alpha(x_i)}]$. For each $\alpha(x_i)$, $(V_{\alpha(x_i)} \setminus U_{\alpha(x_i)}) \cap K$ is a countable set and there exists a countable subset $\Lambda_{\alpha(x_i)}$ of Λ such that $(V_{\alpha(x_i)} \setminus U_{\alpha(x_i)}) \cap K \subseteqq \cup\{U_\alpha : \alpha \in \Lambda_{\alpha(x_i)}\}$. Therefore, we obtain

$$K \subseteqq [\cup_{i\in\mathbb{N}}(\cup\{U_\alpha : \alpha \in \Lambda_{\alpha(x_i)}\})] \cup [\cup_{i\in\mathbb{N}} U_{\alpha(x_i)}]$$

. Converse part follows from the fact that $\gamma_\mu \subseteqq \omega_{\gamma_\mu}$. ■

Corollary 3.3 Let (X,μ) be a strong GTS and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation on μ. Then (X,μ) is γ_μ-Lindelöf if and only if (X, ω_{γ_μ}) is ω_{γ_μ}-Lindelöf.

Theorem 3.4 Let (X,μ) be a strong GTS and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation on μ. Also, let A, B be two subsets of a GTS (X,μ) such that A is γ_μ-Lindelöf relative to X and B is ω_{γ_μ}-closed in X. Then $A \cap B$ is γ_μ-Lindelöf relative to X.

Proof. Let B be an ω_{γ_μ}-closed subset of (X,μ) and $\{U_\alpha : \alpha \in \Lambda\}$ be a cover of $A \cap B$ by γ_μ-open sets of X. For each $x \in A \setminus B$, $x \in X \setminus B \in \omega_{\gamma_\mu}$. Hence, there exists a γ_μ-open set V_x with $x \in V_x$ such that $V_x \cap B$ is a countable set. Since $\{U_\alpha : \alpha \in \Lambda\} \cup \{V_x : x \in A \setminus B\}$ is a cover of A by γ_μ-open sets of (X,μ) and A is γ_μ-Lindelöf relative to X, there exists a countable subcover $\{U_\alpha : \alpha \in \Lambda_1\} \cup \{V_{x_i} : i \in \mathbb{N}\}$, where Λ_1 is a countable subset of Λ such that $A \subseteqq [\cup\{U_\alpha : \alpha \in \Lambda_1\}] \cup [\cup\{V_{x_i} : i \in \mathbb{N}\}]$ and $A \cap B \subseteqq [\cup\{U_\alpha : \alpha \in \Lambda_1\}] \cup [\cup\{B \cap V_{x_i} : i \in \mathbb{N}\}]$. Since $\cup_{i\in\mathbb{N}}(B \cap V_{x_i})$ is a countable set, there exists a countable subset Λ_2 of Λ such that $A \cap [\cup_{i\in\mathbb{N}}(V_{x_i} \cap B)] \subseteqq \cup\{U_\alpha : \alpha \in \Lambda_2\}$. Hence, $\{U_\alpha : \alpha \in \Lambda_1 \cup \Lambda_2\}$ is a countable subcover of $\{U_\alpha : \alpha \in \Lambda\}$ and it covers $A \cap B$. Therefore, $A \cap B$ is γ_μ-Lindelöf relative to X. ■

Corollary 3.5 Let (X,μ) be a strong GTS and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation on μ. Then every ω_{γ_μ}-closed subset of a γ_μ-Lindelöf GTS (X,μ) is γ_μ-Lindelöf relative to X.

Corollary 3.6 Let (X,μ) be a strong GTS and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation on μ. If (X,μ) is γ_μ-Lindelöf and A is γ_μ-closed in X, then A is γ_μ-Lindelöf relative to X.

Proof. This is an immediate consequence of Corollary 3.5, since every γ_μ-closed set is ω_{γ_μ}-closed. ■

Corollary 3.7 Let (X,μ) be a strong GTS and $\gamma_\mu : \mu \to \mathcal{P}(X)$ be an operation on μ. Then the followings are equivalent:
(i) (X,μ) is γ_μ-Lindelöf;

(ii) every proper ω_{γ_μ}-closed subset of (X, μ) is γ_μ-Lindelöf relative to X;
(iii) every proper γ_μ-closed subset of (X, μ) is γ_μ-Lindelöf relative to X.

Proof. (i) $\Rightarrow$ (ii): This is an immediate consequence of Corollary 3.5.

(ii) $\Rightarrow$ (iii): Since every γ_μ-closed set is ω_{γ_μ}-closed, the proof follows from (ii).

(iii) $\Rightarrow$ (i): Let $\{V_\alpha : \alpha \in \Lambda\}$ be a cover of X by γ_μ-open sets of (X, μ). We choose a V_{α_0} such that $X \setminus V_{\alpha_0}$ is a proper subset of X. Then $\{V_\alpha : \alpha \in \Lambda \setminus \{\alpha_0\}\}$ is a cover of $X \setminus V_{\alpha_0}$ by γ_μ-open sets of (X, μ) and $X \setminus V_{\alpha_0}$ is a γ_μ-closed set of X. By (iii), there exists a countable subset Λ_0 of Λ such that $X \setminus V_{\alpha_0} \subseteqq \cup\{V_\alpha : \alpha \in \Lambda_0\}$. Therefore, we obtain $X = [V_\alpha : \alpha \in \Lambda_0 \cup \{\alpha_0\}]$. This shows that (X, μ) is γ_μ-Lindelöf relative to X. ∎

Throughout the rest of the paper, (X, μ) and (Y, λ) will denote GTS's and $\gamma_\mu : \mu \to \mathcal{P}(X)$ and $\beta_\lambda : \lambda \to \mathcal{P}(Y)$ will denote two operations on μ and λ, respectively.

Definition 3.8 A function $f : (X, \mu) \to (Y, \lambda)$ is said to be ω_{γ_μ}-continuous (resp. $(\gamma_\mu, \beta_\lambda)$-continuous) if for each $x \in X$ and each $V \in \beta_\lambda$ containing $f(x)$, there exists $U \in \omega_{\gamma_\mu}$ (resp. $U \in \gamma_\mu$) containing x such that $f(U) \subseteqq V$.

Lemma 3.9 For a function $f : (X, \mu) \to (Y, \lambda)$, the following properties are equivalent:
(i) f is ω_{γ_μ}-continuous;
(ii) $f : (X, \omega_{\gamma_\mu}) \to (Y, \beta_\lambda)$ is $(\omega_{\gamma_\mu}, \beta_\lambda)$-continuous;
(iii) $f^{-1}(V) \in \omega_{\gamma_\mu}$ for every $V \in \beta_\lambda$.

Proof. It is obvious. ∎

Theorem 3.10 If $f : (X, \mu) \to (Y, \lambda)$ is an ω_{γ_μ}-continuous function and K is γ_μ-Lindelöf relative to X, then $f(K)$ is β_λ-Lindelöf relative to Y.

Proof. Let $\{V_\alpha : \alpha \in \Lambda\}$ be a cover of $f(K)$ by β_λ-open sets of Y. Then $\{f^{-1}(V_\alpha) : \alpha \in \Lambda\}$ is a cover of K by ω_{γ_μ}-open sets of X. Since K is γ_μ-Lindelöf relative to X, by Theorem 3.2, there exists a countable subset Λ_0 of Λ such that $K \subseteqq \cup\{f^{-1}(V_\alpha) : \alpha \in \Lambda_0\}$. Hence, we have $f(K) \subseteqq \cup\{V_\alpha : \alpha \in \Lambda_0\}$. Therefore, $f(K)$ is β_λ-Lindelöf relative to Y. ∎

Corollary 3.11 If $f : (X, \mu) \to (Y, \lambda)$ is an ω_{γ_μ}-continuous surjection and (X, μ) is γ_μ-Lindelöf, then (Y, λ) is β_λ-Lindelöf.

Definition 3.12 A function $f : (X, \mu) \to (Y, \lambda)$ is said to be ω_{γ_μ}-closed (resp. $(\gamma_\mu, \beta_\lambda)$-closed) if $f(A)$ is ω_{β_λ}-closed (resp. β_λ-closed) in (Y, λ) for every γ_μ-closed set A of (X, μ).

Lemma 3.13 For a function $f : (X, \mu) \to (Y, \lambda)$, the following properties are equivalent:
(i) f is ω_{γ_μ}-closed;
(ii) $f : (X, \mu) \to (Y, \lambda)$ is $(\gamma_\mu, \omega_{\beta_\lambda})$-closed;
(iii) for each $y \in Y$ and each $U \in \gamma_\mu$ containing $f^{-1}(y)$, there exists an ω_{β_λ}-open set V of Y containing y such that $f^{-1}(V) \subseteqq U$.

Proof. It is obvious. ∎

Theorem 3.14 Let $f : (X, \mu) \to (Y, \lambda)$ be an ω_{γ_μ}-closed function such that $f^{-1}(y)$ is γ_μ-Lindelöf relative to X for each $y \in Y$. If K is β_λ-Lindelöf relative to Y, then $f^{-1}(K)$ is γ_μ-Lindelöf relative to X.

Proof. Let $\{U_\alpha : \alpha \in \Lambda\}$ be any cover of $f^{-1}(K)$ by γ_μ-open sets of (X, μ). For each $y \in K$, $f^{-1}(y)$ is γ_μ-Lindelöf relative to X. Thus, there exists a countable subset $\Lambda_1(y)$ of Λ such that $f^{-1}(y) \subseteqq \cup\{U_\alpha : \alpha \in \Lambda_1(y)\}$. Now, put $U(y) = \cup\{U_\alpha : \alpha \in \Lambda_1(y)\}$. Since f is

ω_{γ_μ}-closed, by Lemma 3.13, there exists an ω_{β_λ}-open set $V(y)$ of Y containing y such that $f^{-1}(V(y)) \subseteqq U(y)$. Since $V(y)$ is ω_{β_λ}-open, there exists $W(y) \in \beta_\lambda$ containing y such that $W(y)\setminus V(y)$ is a countable set. For each $y \in K$, we have $W(y) \subseteqq [W(y)\setminus V(y)] \cup V(y)$ and hence

$$f^{-1}(W(y)) \subseteqq f^{-1}[W(y) \setminus V(y)] \cup f^{-1}(V(y)) \subseteqq f^{-1}[W(y) \setminus V(y)] \cup U(y).$$

Since $W(y) \setminus V(y)$ is a countable set and $f^{-1}(z)$ is γ_μ-Lindelöf relative to X for each $z \in Y$, there exists a countable set $\Lambda_2(y)$ of Λ such that $f^{-1}([W(y)\setminus V(y)] \cap K) \subseteqq \cup\{U_\alpha : \alpha \in \Lambda_2(y)\}$ and hence

$$f^{-1}(W(y) \cap K) \subseteqq \cup\{U_\alpha : \alpha \in \Lambda_2(y)\} \cup \{U_\alpha : \alpha \in \Lambda_1(y)\}.$$

Since $\{W(y) : y \in K\}$ is a cover of K by β_λ-open sets of Y and K is β_λ-Lindelöf relative to Y, there exist countable number of points of Y, say, $y_1, y_2, ..., y_n, ...$ such that $K \subseteqq \cup\{W(y_i) : i \in \mathbb{N}\}$. Therefore, we obtain $f^{-1}(K) \subseteqq \cup\{f^{-1}(W(y_i) \cap K) : i \in \mathbb{N}\} \subseteqq \cup_{i\in\mathbb{N}}[\{\cup U_\alpha : \alpha \in \Lambda_2(y_i)\} \cup \{\cup U_\alpha : \alpha \in \Lambda_1(y_i)\}] = \cup\{U_\alpha : \alpha \in \Lambda_1(y_i) \cup \Lambda_2(y_i), i \in \mathbb{N}\}$. This shows that $f^{-1}(K)$ is γ_μ-Lindelöf relative to X. ■

Corollary 3.15 Let $f : (X, \mu) \to (Y, \lambda)$ be an ω_{γ_μ}-closed function such that $f^{-1}(y)$ is γ_μ-Lindelöf relative to X for each $y \in Y$. If (Y, λ) is β_λ-Lindelöf, then (X, μ) is γ_μ-Lindelöf.

Corollary 3.16 Let $f : (X, \mu) \to (Y, \lambda)$ be an ω_{γ_μ}-continuous and ω_{γ_μ}-closed surjection such that $f^{-1}(y)$ is γ_μ-Lindelöf relative to X for each $y \in Y$. Then (X, μ) is γ_μ-Lindelöf if and only if (Y, λ) is β_λ-Lindelöf.

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